Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

Let me begin by reviewing the criteria used to select solutions for publication.

Responses are collected as they arrive, with no regard to their date of arrival.

When writing the column, I first eliminate incorrect and confusing responses. Then I favor solutions that are easy to typeset and those from first-time respondents.

Problems

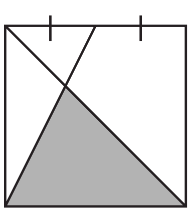
N/D 1. Larry Kells wants you to find a bridge hand in which North-South can make some contract against best defense, even though neither of them has a card higher than a 9.

N/D 2. Dave Blackston tells us that number theorists call a positive integer p-smooth if all its prime factors are at most p. The very large integer $N = 6^{100,000}$ is clearly 3-smooth. Blackston asks for the smallest 3-smooth number that is larger than N.

N/D 3. Our last regular problem this issue is one John Astolfi calls an "old-time safari puzzle." The year was 1888, and famed explorer Sir Rigglesworth was stymied. He wished to traverse on foot a totally barren desert that would take a person six days to cross. But a person could carry only four days' rations of food and water. Fortunately, two of his bearers, Al and Zack, put their heads together and came up with a plan to get Sir Rigglesworth successfully across. How did they do it?

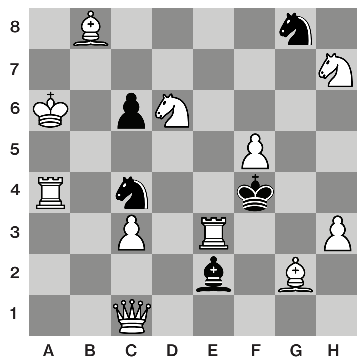
Speed department

Sorab Vatcha asks: How big is the shaded triangle inside the unit square?



Solutions

J/A 1. We begin with a rather unusual chess problem from Sorab Vatcha. Most mate-in-two chess problems have a unique solution; some have two or three variations. The following problem has many variations, perhaps an all-time record number. White is to play and win in two moves. Find all solutions.



Michael Branicky found the following 25 solutions, all beginning with Be4. Specifically

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1. Be4   Ke5        1. Be4   Ne7        1. Be4   Na3+
2. NxN   mate       2. NxN   mate       2. Nc4   mate

1. Be4   Ke5        1. Be4   Nf6        1. Be4   Na5+
2. Nf7   mate       2. NxN   mate       2. Nc4   mate

1. Be4   Bd1        1. Be4   Nh6        1. Be4   Nb2+
2. NxN   mate       2. NxN   mate       2. Nc4   mate

1. Be4   Bd3        1. Be4   Na3+       1. Be4   Nb6+
2. NxN   mate       2. Nb5   mate       2. Nc4   mate

1. Be4   Bf1        1. Be4   Na5+       1. Be4   Nd2+
2. NxN   mate       2. Nb5   mate       2. Nc4   mate

1. Be4   Bb3        1. Be4   Nb2+       1. Be4   NxN
2. NxN   mate       2. Nb5   mate       2. Bd3   mate

1. Be4   Bg4        1. Be4   Nb6+       1. Be4   NxR
2. NxN   mate       2. Nb5   mate       2. Nb5   mate

1. Be4   Bh5        1. Be4   Nd2+       1. Be4   Neb
2. NxN   mate       2. Nb5   mate       2. Rd3   mate

1. Be4   c5
2. NxN   mate
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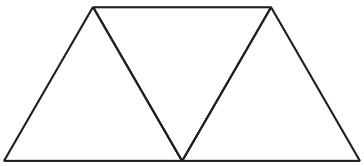
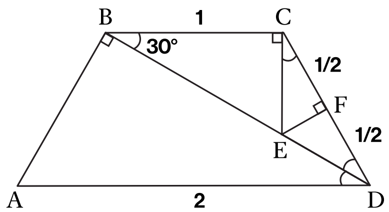
J/A 2. The late Dick Hess sent us the following problem, which he attributed to Bob Wainwright. The diagram below shows an equilateral trapezoid constructed from three equilateral triangles. You are to divide the figure into four similar pieces of three different sizes (i.e., exactly two pieces are congruent).



Jeffrey Schenkel and Thomas Kelley independently found the following decomposition.

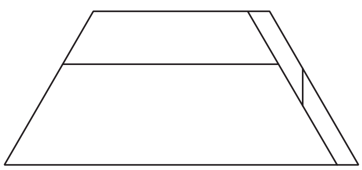


Let the equilateral triangles have side length 1. Form BD and note, by the law of cosines, $\overline{BD} = \sqrt{3}$.

Since $\overline{AB}^2 + \overline{BD}^2 = \overline{DA}^2$, angle ABD = 90°. Then angle DBC = angle BDA = angle BDC = 30°. Form CE perpendicular to BC at C. Then angle ECD = 30°. Form EF perpendicular to CD at F and note that CF = FD.

This creates four similar 30-60-90 triangles: ABD with hypotenuse $\overline{AD} = 2$, ECB with hypotenuse $\overline{EB} = 2/\sqrt{3}$, EFC with hypotenuse $\overline{EC} = 1/\sqrt{3}$, and ECB with hypotenuse $\overline{ED} = 1/\sqrt{3}$.

A different solution, submitted independently by Greg Muldowney and Marc Strauss, looks like this:



Muldowney's full solution is on the Puzzle Corner website.

John Chandler, starting with the Schenkel/Kelley diagram, constructs X the intersection of the diagonals AC and BD, and Y the midpoint of BC. His four pieces are triangles ACD, ABX, BXY,and CXY. Stanley Liu has results on a similar problem.

J/A 3. Spyros Kinnas has three variations on the "floating ice problem." If ice floating on water melts, the level of the water does not change. What happens to the water level when the ice melts in each of the following situations? In all three cases the ice is floating.

1. There are pockets of air trapped inside the ice (ignore the weight of the trapped air).
2. There is a solid inside the ice (or on top) with density less than that of water (e.g., wood).
3. There is a solid inside the ice (or on top) with density more than that of water (e.g., steel).

The following solution is from Loren Bonderson.

A floating object is in equilibrium with two equal forces acting on it, its weight and a buoyant force. By Archimedes' Principle the buoyant force is equal to the weight of the displaced water. Thus, the magnitude is equal to the product of the volume of displaced water and the density of water.

Case 1. The weight of the floating object equals the weight of the ice, so the weight of the displaced water equals the weight of the ice. When the ice melts, the resulting water will have the same volume as the displaced water. The water level will then not change.

Case 2. The initial buoyant force equals the combined weights of the ice and the wood. Then the ice melts and the water from the melt exactly fills the displaced water that produced the portion of the buoyant force needed to float only the ice. However, the wood still floats, and the displaced water volume necessary to produce the buoyant force to float the wood remains the same as it was initially. Again, there's no change in the water level.

Case 3. The initial buoyant force equals the combined weights of the ice and the steel. Then the ice melts and the melt equals the displaced water volume that produced the buoyant force needed to float the ice. The steel sinks and the volume of water it displaces is the same as the volume of the steel. This maximum volume of displaced water is less than the original volume that originally allowed the steel to float. Thus, the water level decreases.

Better late than never

2015 S/O SD. Michael Barr has been studying the infinite tower of exponentials $x\hat{}\ x\hat{}\ x\hat{}$ ... and conjectures that the largest x for which the tower converges is $e^{1/e}$.

2018 M/J 3. Henry Hodara has an "equationless" solution that is now on the Puzzle Corner website.

Other responders

Responses have also been received from J. d'Almeida, R. DeJong, E. Field, J.-P.Garric, J. Harmse, J. Larsen, R. Luise, T. Mita, R. Morgen, A. Ornstein, A. Stern, and D. Worley.

Solution to speed problem

1/3. The small triangle is similar to the shaded; hence its height is 1/2 that of the shaded, and thus the shaded triangle has height 2/3.