Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

During my 50th MIT reunion in June 2017, I was interviewed on camera by the Alumni Association. In May, the video "The Puzzle Guy" appeared on the mit.edu home page. (As the editor's note in July/August reported, you can also see it at technologyreview.com/puzzle-guy.) When I first viewed the video this spring, it seemed considerably more organized than I remembered my performance at the interview. A transcript of the raw text confirmed that the final result is *very* considerably more organized.

More recently my beautiful wife, Alice, received a major award from the National Psoriasis Foundation for her foundational research showing that psoriasis is fundamentally an immunological disease with dermatological signs and symptoms. She introduced the use of biologics for treating it. Today, many modern treatments for psoriasis are biologics, highlighting the importance of Alice's discovery.

Problems

S/O 1. We start with a bridge problem from Larry Kells, who wants you to find a hand so that South makes seven spades against best defense, despite one opponent holding the KJ975 of spades and a side KJ. The other opponent has two KJs as well. Finally, South cannot make any other grand slam against best defense.

S/O 2. Our second problem is from David Singmaster's book *Problems for Metagrobologists*. A father is traveling with his N children, named 1, 2, … , N. Child i is one year older than child $i + 1$. Each child is happy to be with any child other than a sibling just one year younger or older. Children i and $i + 1$ cannot be alone together. The family comes to a river, and they find a rowboat that can hold only two people. Only the father and the child named 1 (i.e., the oldest child) can row. What is the smallest number of crossings that will get the entire family across the river?

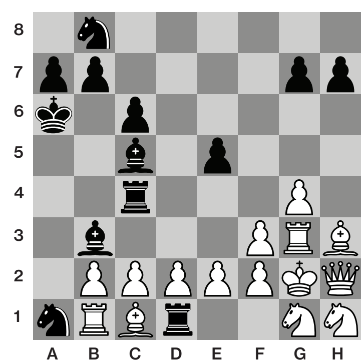
S/O 3. The NFL season should be in full swing by the time you read this problem from Robert Bird, who asks you to find the minimum initial velocity (ignoring friction) and the corresponding optimum angle for a football kicked from the ground so that it will strike the crossbeam on the goalpost. You are given x, the horizontal distance to the goalpost; y, the height of the crossbeam; and g, the acceleration of gravity.

Speed department

Sorab Vatcha wants to know the key to integrating $\int \frac{1}{in}\, d(in)$ on a computer.

Solutions

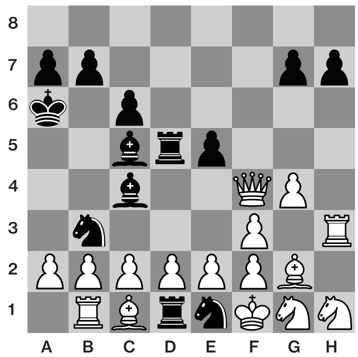
M/J 1. Timothy Chow asks whether the position shown in the following diagram can occur in a legal game. That is, can the position be reached from the initial state by a sequence of legal moves?



This problem makes me realize how good it is that my job is to edit the column, not produce solutions. Dave Mellinger and several others, however, do not suffer my limitations.

Mellinger writes: "Good puzzle! At first I thought this position was impossible to reach, since what could White's last move have been? Then I thought it must involve a Black pawn reaching a1 and becoming the knight there (with White's last move a1-a2), but that's precluded by the arrangement of Black's remaining pawns and the fact that White has 15 pieces on the board, allowing only one capture for Black. Finally I realized that Black's final move must have been the capture of White's missing pawn.

"So: Players move until the position is the one shown in the figure below. If at any time Black needs extra moves, White can simply move the rook between a1 and b1; if White needs extra moves, Black can, among many options, move the rook between d4 and d5. Black's rook on d1 got in behind White's pawns via the h-file, which White can empty out into the middle of the board as needed. White's bishop need not leave the confines of the f1-h3 diagonal to do this, and White's king need not go any farther than g3."



From the figure we proceed as follows.

1. Rg3	Rd4	8. Rb1	Nh5	15. Pa5	Kb5
2. Bh3	Rd5	9. Ra1	Nf6	16. Pa6	Kxa6
3. Rg2	Rd4	10. Rb	Nd7		
4. Qh2	Rd5	11. Ra1	Nb8		
5. Rg3	Rd4	12. Rb1	Na1		
6. Kg2	Nd3	13. Pa3	Bb3		
7. Ra1	Nf4	14. Pa4	Rc4		

M/J 2. In honor of the famous Japanese puzzle inventor Nob Yashigahara, Robert High gives us this cryptarithmetic problem.

$$NOB^X + Y \times NOB = PUZZLES$$

This problem is unusual in using exponentiation and in permitting X and Y to take the value of any digit between 0 and 9, *including* values assumed by the other letters. The uniqueness of the solution, High believes, demonstrates that Nob has a unique relationship to puzzles.

All solvers used some form of searching and obtained

$$172^3 + 3 \times 172 = 5{,}088{,}964$$

One technique for limiting the search, used by Phillip Davis and others, was to note that the lowest *PUZZLES* is 1,022,345 and the highest *PUZZLES* is 9,877,654. To be in this range, a three-digit *NOB* must be cubed and in the range 101 to 214. Also limiting: for *NOB* less than 126, $N = P = 1$, which is not allowed. Therefore the allowable range is 126 to 214; each of those 89 possibilities has 10 possible *Y*s.

If you solve in addition $HIGH^X + Y \times HIGH = PUZZLES$, you will see that the problem's author also has a unique relationship to puzzles: $3{,}073^2 + 4 \times 3{,}073 = 9{,}455{,}621$

Burgess Rhodes also solves $H^I + G^H = HIGH$: $2^5 \times 9^2 = 2{,}592$.

M/J 3. I received several fine inductive proofs that $P(n, N)$, the probability that the nth passenger out of N will get his or her assigned seat, is $1/N$ if $n = 1$ and $(N - n + 1)/(N - n + 2)$ otherwise. Charles Wampler notes that $P(2, n) = (N - 1)/N$ as desired and then refers to passenger i's assigned seat as seat i. By induction he assumes that the formula holds for $n < m$ and proceeds as follows for $n = m$.

If passenger 1 takes seat 1 or any seat greater than m, a total of $N - m + 1$ possibilities, then passengers 2 to m all take their own seats. Alternatively, if passenger 1 takes seat $2 \le k < m$, then passengers 2 through $k - 1$ all take their own seats. The situation is then equivalent to starting with a fresh plane with $N - (k - 1)$ seats and passengers, with the former passengers k through N renumbered downward, so that original passenger m is now passenger $m - (k - 1)$ out of $N - (k - 1)$. Tallying all the possibilities gives

$$P(m, N) = \frac{N - m + 1}{N} + \frac{1}{N}\sum_{k=2}^{m-1} P(m - (k - 1), N - (k - 1))$$

Applying the induction hypothesis, we find that each summand is equal and that, as desired,

$$P(m, N) = \frac{N - m + 1}{N} + \frac{m - 2}{N}\,\frac{N - m + 1}{N - m + 2} = \frac{N - m + 1}{N - m + 2}$$

Better late than never

2017 N/D 3. Burgess Rhodes has a modest generalization and some interesting related properties. See the Puzzle Corner website for details.

2018 J/F 1. Jim Larsen's correct solutions were somehow butchered between my receiving them and the final publication in May/June. I have placed the original correct solutions on the website. I thank several readers, especially Larsen, for pointing this out.

Other responders

Responses have also been received from A. Andersson, T. Chow, D. Emmes, P. Davis, R. DeJong, D. Foxvog, B. Frederickson, C. Gains, J. Grossman, R. Guldi, J. Harmse, H. Hodara, J. Larsen, Z. Levine, D. Loeb, J. Marcou, J. Marlin, M. McComb, D. Mellinger, T. Mita, R. Morgen, G. Muldowney, A. Ornstein, S. Peters, R. Schooler, E. Signorelli, T. Sim, M. Smith, A. Stern, R. Stern, T. Royanovskii, T. Tamura, S. Vatcha, B. Wake, C. Wampler, K. Wise, and D. Worley.

Solution to speed problem

Log in.