Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

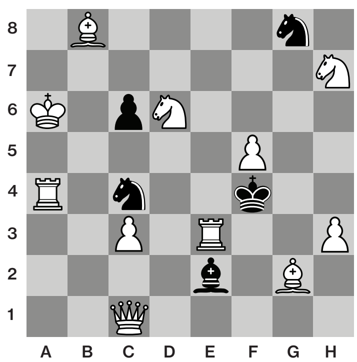
Editor's note: To hear Allan Gottlieb reflect on his 50-plus years as editor of Puzzle Corner, visit technologyreview.com/puzzle-guy.

I was saddened to receive the following message from Ermanno Signorelli: "Dick Hess '57, a prolific contributor to the Puzzle Corner, passed away on February 10, 2016 (see page 30 of MIT News in the March/April issue)."

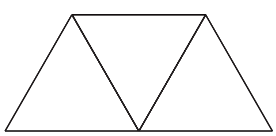
He will be missed.

Problems

J/A 1. We begin with a rather unusual chess problem from Sorab Vatcha. Most mate-in-two chess problems have a unique solution; some have two or three variations. The following problem has many variations, perhaps an all-time record number. White is to play and win in two moves. Find all solutions.



J/A 2. Dick Hess had sent us the following problem, which he attributed to Bob Wainwright. The diagram below shows an equilateral trapezoid constructed from three equilateral triangles. You are to divide the figure into four similar pieces of three different sizes (i.e., exactly two pieces are congruent).



J/A 3. Spyros Kinnas has three variations on the well-known "floating ice problem":

We all know that if ice floating on water melts, the level of the water does not change. What happens to the water level when the ice melts in each of the following situations? In all three cases the ice is floating.

1. There are pockets of air trapped inside the ice (ignore the weight of the trapped air).
2. There is a solid inside the ice (or on top) with density less than that of water (e.g., wood).
3. There is a solid inside the ice (or on top) with density more than that of water (e.g., steel).

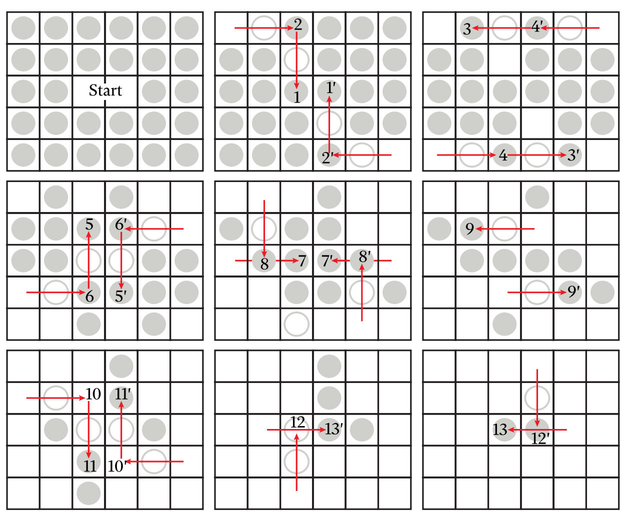
Speed department

John Prussing has a discount coupon worth y dollars for an item that costs $x > y$ dollars when purchased as an individual package and $2x$ dollars for a three-pack. He wonders if, using the coupon, he will always pay a smaller per-item price buying the three-pack.

Solutions

M/A 1. Greg Muldowney sent us a solution to Rocco Giovanniello's wink problem that is nearly symmetric with respect to the vertical center line. He included a graphical depiction as well as the more familiar "Cartesian" description.

In the solution diagrammed below, the first 22 moves are pairs of independent point-symmetric jumps numbered 1/1′, 2/2′, . . . , 11/11′ with jump n ending to the left of the vertical center line and n' to the right. Four pairs are vertical jumps and seven are horizontal, and in all jumps the wink moves from its resident square toward the vertical or horizontal center line of the array. The final four jumps are also point-symmetric pairs, but are executed in the order 12-13′-13-12′.



In index notation where i,j denotes the ith row from the top and jth column from the left, the foregoing sequence is:

1/1′, 2/2′:	1,3 → 3,3	5,4 → 3,4	1,1 → 1,3	5,6 → 5,4
3/3′, 4/4′:	1,4 → 1,2	5,3 → 5,5	5,1 → 5,3	1,6 → 1,4
5/5′, 6/6′:	4,3 → 2,3	2,4 → 4,4	4,1 → 4,3	2,6 → 2,4
7/7′, 8/8′:	3,1 → 3,3	3,6 → 3,4	1,2 → 3,2	5,5 → 3,5
9/9′:	2,4 → 2,2	4,3 → 4,5		
10/10′, 11/11′:	2,1 → 2,3	4,6 → 4,4	2,3 → 4,3	4,4 → 2,4
12/13′, 13/12′:	5,3 → 3,3	3,2 → 3,4	3,5 → 3,3	1,4 → 3,4

M/A 2. Another "logical hat" problem from the late Dick Hess. Consecutive integers are chosen from 1, 2, 3, ... One is written on the hat of logician A and the other is written on the hat of logician B. Each logician sees the other's hat but not his or her own. Each is error-free in reasoning and knows the situation. They say in turn

A1: "I don't know my number."

B1: "I don't know my number."

A2: "I now know my number."'

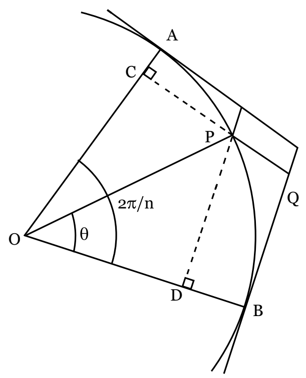
What numbers are possible for A and B?

The following analysis from Paul Burstein concludes that there are two solutions (A = 3, B = 2 and A = 4, B = 3). A number of readers agree with Burstein. Another group feels that only A = 3, B = 2 is possible and a third group only A = 4, B = 3. We suspect that the action occurs close to the 1, 2, 3 end of the scale; otherwise, triples of the form n, $n - 1$, $n + 1$ will never allow A to know the answer after seeing B's number and hearing B's reply. There may be a more elegant way to do this, but I don't see one. So, here's the complete Henning Mankell treatment.

1. A does not knows his *[or her or their–Ed.]* number: The only number that A could see on B's hat and know his own number is 1, but A does not know. So B's number must be >1.
2. B does not know his number:
   - A's number is not 1. (If A's number were 1, B's number must be 2, which is a no-go.)
   - A's number must be greater than 2. (If A's number is 2, and everybody knows that B's number is not 1, then B would deduce that his own number must be 3. But B does not know.)
3. A knows his number:
   - If B's number is 2, then A will know that his own number must be 3.
   - If B's number is 3, then A will know that his own number must be 4. (The other choice, A = 2, would have resulted in B's knowing his own number.)
   - If B's number is 4, then A's number could be 3 or 5. If A's number is 3, then B knows his own number is 2 or 4. If A's number is 5, then B knows his own number is 4 or 6. In either case, B doesn't know. So A would not know either, i.e., whether A's own number was 3 or 5. So B cannot be 4.
   - If B's number exceeds 4, we get a similar contradiction as when B's number is 4.
4. So the largest possible value for B is 3, and there are two solution: A = 3, B = 2 and A = 4, B = 3.

M/A 3. Our final regular problem is a generalization by Larry Stabile of problem 2 from the 2016 September/October issue. Given an n-sided figure (partially shown here for $n = 5$) with the radius of the inscribed circle equal to R, draw radii to two adjacent points of tangency between the polygon and the inscribed circle. Find the area of the shaded region as given by the boundary formed by the edges of the polygon and parallel lines from the point P intersecting the edges, over the domain $\theta \epsilon [0, 2\pi/n]$.

The following solution is from Tony Yen.



As the figure above shows, the area of the parallelogram in question is $\overline{AC} \cdot \overline{PQ}$

$$\overline{AC} = R - R\cos(2\pi/n - \theta) : \overline{BD} = R - R\cos\theta \; ; \; \overline{PQ} = \frac{\overline{DB}}{\sin(2\pi/n)}$$

$$\overline{AC} \cdot \overline{PQ} = \frac{R^2[1-\cos(2\pi/n-\theta)][1-\cos\theta]}{\sin(2\pi/n)}$$

Better late than never

3 2017 N/D. David Chandler and John Makhoul offered improvements; each is on the Puzzle Corner website.

SD 2018 M/A. Sorab Vatcha notes that 1,729 is known as the Hardy-Ramanujan number and the second taxicab number.

Other responders

Responses have also been received from J. Arsenault, T. Barrows, R. Bird, P. Davis, R. DeJong, D. Detlefs, P. Groot, J. Hardis, J. Harmon, P. Kramer, J. Larson, Z. Levine, N. Markovitz, T. Mita, S. Nason, P. Paternoster, J. Prussing, B. Rhodes, L. Schaider, A. Shuchat, E. Signorelli, S. Sperry, J. Weisman, N. Williams, P. Winterfeld, D. Worley, and J. Wrinn.

Solution to speed problem

No. If $y > x/2$, the cost for the single item $x - y$ is less than the cost per item for the three-pack $(2x - y)/3$.