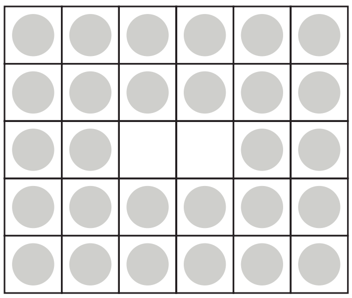
It has been a few years since I reviewed the criteria used to select solutions for publication. Let me do so again.

As responses arrive, they are simply put together in neat piles, with no regard to their date of arrival. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I try to select a solution that supplies an appropriate amount of detail and includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred, as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or (especially) sent by e-mail, since these simplify typesetting.

Problems

M/A 1. Our wink-meister Rocco Giovanniello has a new problem with a twist: this time two winks are initially missing. Specifically, consider the following 5×6 rectangle mostly filled with winks, as shown. Each move is a horizontal or vertical jump of one wink over another, landing on an empty square. The jumped-over wink is removed. The goal is to remove all the winks save two, which are in the initially empty two squares.



M/A 2. Another "logical hat problem" from Dick Hess.

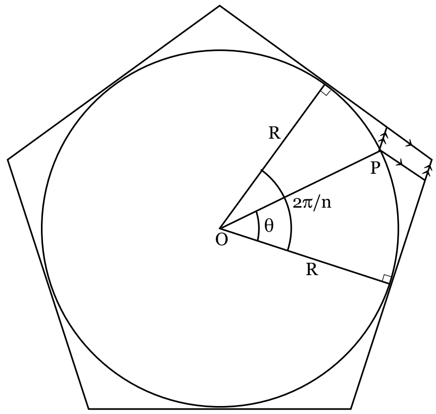
Consecutive integers are chosen from 1,2,3, ... One is written on the hat of logician A and the other is written on the hat of logician B. Each logician sees the other's hat but not his or her own. Each is error-free in reasoning and knows the situation. They say in turn

A1: "I don't know my number."
B1: "I don't know my number."
A2: "I now know my number."

What numbers are possible for A and B?

M/A 3. Our final regular problem is a generalization by Larry Stabile of problem 2 from the September/October 2016 issue.

Given an n-sided regular polygon (shown here for $n = 5$) with the radius of the inscribed circle equal to R, draw radii to two adjacent points of tangency between the polygon and the inscribed circle. Find the area of the shaded region as given by the boundary formed by the edges of the polygon and parallel lines from the point P intersecting the edges, over the domain $\theta \,\varepsilon\, [0, 2\pi/n]$.



Speed Department

Can you determine what Sorab Vatcha's favorite number is? It has the following properties.

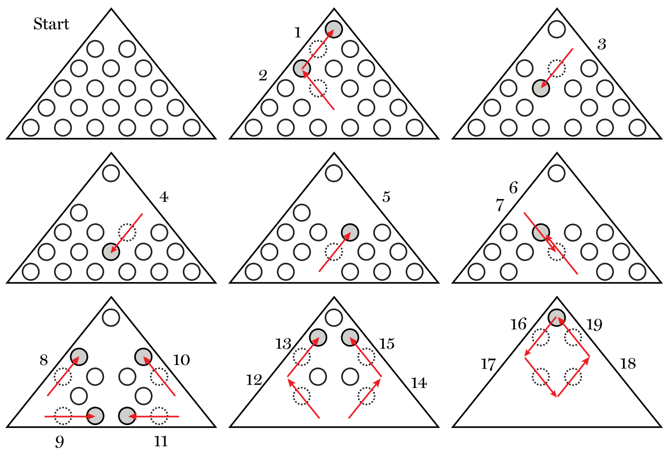
1. It is the smallest number equal to the sum of two positive cubes in two ways.
2. It is the product of a palindrome pair.
3. Its three prime factors are the first, third, and fifth entries in a sequence of five consecutive prime numbers.
4. The sum of its digits equals its largest prime factor.

Solutions

N/D 1. Rocco Giovanniello offers a six-row triangular grid of 20 winks, with the apex (row 1) empty and rows 2 through 6 filled, to be solved in 19 diagonal or horizontal jumps with the jumped-over wink removed and the last wink at the apex.

Greg Muldowney sent us the following solution, including "action shots" of the wink moves.

In the solution diagrammed below, jumps 1 through 3 free up space at the symmetry line, and all jumps thereafter are mirror-image pairs. Jumps 4/5 and 6/7 open symmetric positions in rows 3 and 5, enabling full vacation of the lower corners in jumps 8/9 and 10/11. Jumps 12/13 and 14/15 empty rows 5 and 6 and refill row 2. This sets up a "home run" (jumps 16 through 19) in which the apex wink traverses the four sides of a diamond and returns to its starting point. Thus the first wink moved is also the last.



In index notation where i,j denotes the ith row from the top and jth position from the left, the foregoing sequence is:

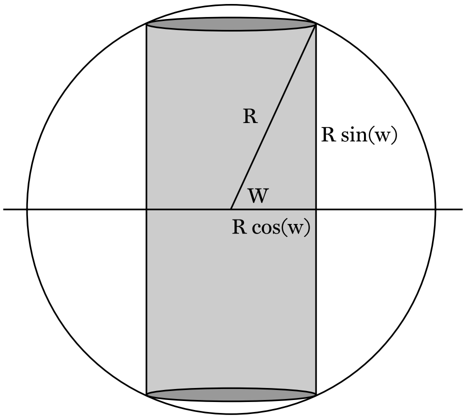
1 – 3:	3,1 → 1,1	5,3 → 3,3	2,2 → 4,2	
4 – 7:	3,3 → 5,3	6,3 → 4,3	3,1 → 5,3	6,4 → 4,2
8 – 11:	5,1 → 3,1	5,5 → 3,3	6,1 → 6,3	6,6 → 6,4
12 – 15:	6,3 → 4,1	4,1 → 2,1	6,4 → 4,4	4,4 → 2,2
16 – 19:	1,1 → 3,1	3,1 → 5,3	5,3 → 3,3	3,3 → 1,1

Except for moves along the bottom row (9 and 11), all jumps are diagonal.

N/D 2. Sorab Vatcha has a sphere of radius R and wonders what is largest volume cylinder that can fit inside.

David Dewan notes that solving the problem brought back memories of Professor George Thomas. Dewan's detailed solution and clear diagram follow.

Draw a line at angle w from the center of the sphere to the intersection of the cylinder and sphere.



The cylinder's radius is $R\cos(w)$ and its height is $2R\sin(w)$. Hence its volume is $2\pi R^3\cos^2(w)\sin(w)$.

The volume is 0 at w = 0 (flatter than a pancake), 0 at w = $\pi/2$ (skinnier than a straw), and maximum somewhere in between, where the derivative of the volume = 0.

Ignoring the constants ($2\pi R^3$), d(volume)/dw = d($\cos^2$(w) sin(w))/dw = $\cos^3(w) - 2\sin^2(w)\cos(w)$.

Setting d (volume)/dw = 0 and simplifying gives cos(w) = $\sqrt{2/3}$ and sin(w) = $\sqrt{1/3}$, hence the volume is $4\pi R^3/3\sqrt{3}$.

N/D 3. Generalization: Consider functions $f(x, y)$ with the property (P)

$$\min\{x, y\} < f(x, y) < \max\{x, y\},\ x,y > 0,\ x \neq y.$$

Such functions produce output values strictly between their input values x and y. The arithmetic mean $(x + y)/2$, geometric mean $\sqrt{xy}$, and harmonic mean $2/(1/x + 1/y)$ are functions with this property.

Now consider any two functions f and g with property (P). With positive initial values a_0 and b_0, define iteration $a_{i+1} = f(a_i, b_i)$ and $b_{i+1} = g(a_i, b_i)$, $i = 0, 1, \ldots$

Bounds on output values are $\min\{a_i, b_i\} < a_{i+1} < \max\{a_i, b_i\}$ and $\min\{a_i, b_i\} < b_{i+1} < \max\{a_i, b_i\}$

from which determine $\min\{a_i, b_i\} < |a_{i+1} - b_{i+1}| < \max\{a_i, b_i\}$

Thus $$\frac{|a_{i+1} - b_{i+1}|}{|a_i - b_i|} = \rho_i < 1.$$

Consequently, $$|a_{i+1} - b_{i+1}| = \left(\prod_{j=0}^{i} \rho_j\right) |a_0 - b_0$$

and because $$\lim_{i\to\infty} \prod_{j=0}^{i} \rho_j = 0\ ,$$

$$\lim_{i\to\infty} a_{i+1} = \lim_{i\to\infty} b_{i+1} = L.$$

Conclusion: For any two functions with property (P) in the calculations of a_{i+1} and b_{i+1} above, the sequences $\{a_i \mid i = 0, 1,\ldots\}$ and $\{b_i \mid i = 0, 1,\ldots\}$ have a common limit $L(a_o, b_o)$.

Limiting values $L(a_o, b_o)$ with $a_o = 1$ and the arithmetic and geometric mean functions are shown on the website. (Note: Convergence is fast.)

Better Late Than Never

2017 M/J 3. Ken Wise has found a way to generalize the solution to require only geometry and no algebra.

J/A 2. The solution posted on the website is due to Lou Metzger.

J/A 3. Burgess Rhodes feels that "to move from the seemingly impossible to a simple solution, the four mathematicians agree to operate in base 3." Then if each mathematician chooses the number 3 (which is written 10_3 in base 3) and performs all arithmetic in base 3, the conditions of the problem are satisfied.

$$(10_3)^{k-2} + (10_3)^{k-2} + (10_3)^{k-2} = 10_3 \cdot (10_3)^{k-2} = (10_3)^{k-1}$$

has k digits, k = 20, 22, 26, and 34.

N/D 1. Todd Tamura and his son found a promotion-free solution with 107 moves.

Other Responders

Responses have also been received from P. Davis, R. Downey, D. Foxvog, B. Frederickson, J.P. Garric, S. Kanter, T. Kurtz, J. Larsen, T. Mattick, L. Metzger, T. Mita, R. Morgen, B. Olness, A. Ornstein, J. Prussing, H. Safter, G. Schaffer, E. Signorelli, C. Swift, P. van der Varst, and D. Worley.

Proposer's Solution to Speed Problem

$1{,}729 = 1^3 + 12^3 = 9^3 + 10^3 = 19 \times 91 = 7 \times 13 \times 19$. 1+7+2+9 = 19

Send problems, solutions, and comments to Allan Gottlieb, New York University, 60 Fifth Avenue, Room 316, New York, NY 10011, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.