This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 8) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2017 yearly problem is in the "Solutions" section.

Problems

Y2018. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 8 exactly once each, along with the operators +, -, × (multiplication), / (division), and ^ (exponentiation)? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 8 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Larry Kells knows (because we ran the problem earlier) that even with 26 high-card points, a defender cannot be assured of defeating 7 no-trump (i.e., there is a distribution where, with best play, declarer makes 7 no-trump). Larry wonders what are the corresponding number of points for 6, 3, and 1 no-trump.

J/F 2. Michael Auerbach has an "ominoes" question. Clearly with one 1x1 square tile, only one unique shape can be formed. The same is true with two such tiles (only one domino). There are only two triominoes, a 3x1 and an L-shape (we consider two shapes the same if one can be turned into the other by rotation, translation, or flipping it over). Michael knows there are five tetrominoes and 12 pentominoes. He asks for the number of hexominoes.

Speed Department

Ermanno Signorelli reports that "Adam is looking only at Beatrice, but Beatrice is looking only at Calvin. Adam is married but Calvin is not." Ermanno wonders if a married person is looking at an unmarried person.

Solutions

Y2017. David Cane sent us the following solution and notes that for 23 solutions a leading 0 would give a "better" solution. However, I have never permitted leading 0s. We use "^" to indicate exponentiation.

1 = 217^0	12 = (20 − 1) − 7	24 = 2 × 7 + 10
2 = 2 + 0 × 17	13 = 12 + 7^0	26 = 20 − 1 + 7
3 = 20 − 17	14 = 20 + 1 − 7	27 = 20 + 1 ×7
4 = 2 × 7 − 10	15 = 17 − 2 + 0	28 = 20 + 1 + 7
5 = 2 + 10 − 7	16 = 17 − 2^0	30 = 210/7
6 = 7 − 21^0	17 = 27 − 10	34 = (2 + 0) × 17
7 = 21× 0 + 7	18 = 17 + 2^0	35 = 1/2 × 70
8 = 21^ 0 + 7	19 = 2 + 0 + 17	36 = 1 + 70/2
9 = (10 − 7)^2	20 = 20 − 1 − 7	37 = 20 + 17
10 = 2 + 0 + 1 + 7	21 = 20 − 1 × 7	39 = 7^2 − 10
11 = 12 − 7^0	22 = 21 + 7^0	48 = 0 − 1 + 7^2

49 = 70 − 21	67 = 70 − 2 − 1	84 = 0 + 12 × 7
50 = 10 (7 − 2)	68 = 10 × 7 − 2	85 = 170/2
51 = 71 − 20	69 = 0 − 2 + 71	90 = (2 + 7) 10
56 = (10 − 2) × 7	70 = 1^2 × 70	91 = 20 + 71
58 = 70 − 12	71 = 2 × 0 + 71	93 = 10^2 − 7
59 = 10 + 7^2	72 = 2^0 + 71	95 = 102 − 7
62 = 72 −10	73 = 2 + 0 + 71	
64 = 2^(0 − 1 + 7)	82 = 12 + 70	

S/O 1. Larry Kells is now interested in what I might call "lazy bridge," where the declarer need only play legally to succeed. Specifically, he asks if it is possible to construct a deal that guarantees North and South will make a game in any suit or no-trump regardless of the line of play by both sides. He also wonders: What is the largest number of tricks for which a deal exists such

	♠ A K Q J	
	♥ A K Q	
	♦ A K Q	
♠ x x x	♣ A K Q	♠ x x x
♥ x x x		♥ x x x x
♦ x x x		♦ x x x
♣ x x x x	♠ x x x	♣ x x x
	♥ x x x	
	♦ x x x x	
	♣ x x x	

that North and South are assured of that many tricks no matter what the contract is or how poorly the declarer plays?

I guess Jim Larsen is as lazy as anyone. His solution follows.

All hands having 4-3-3-3 distributions and North having AKQ in each suit with a jack in one will result in North and South (the declarer) making game in any suit or no-trump regardless of the line of play by both sides. The largest number of tricks assured regardless of the contract is 11.

The limiting case for "legally, regardless of the line of play" is absolute worst-case play by the declarer and best-case play by the defense. Declarer will make the most disadvantageous lead, throw winner on winner, trump winners, and hold back trump when he could legally trump, while Defense plays optimally. To be legal, however, all must follow suit when they have a card in the suit led.

No-trump is straightforward, and many hands can be constructed that guarantee 13 tricks. All that is required is for Declarer to have all the aces and sequences of top controlling cards that flow from them. The challenge is when a trump is declared.

One of Declarer's hands must have all the top cards, but the key to his/her success lies in tightly balanced distribution and having no entries to the other of Declarer's hands. If either Declarer or Defense had long and short suits, more trumping opportunities would exist. Which hand holds the long card in each suit is arbitrary.

Working from the long cards in each hand being N: spades, E: hearts, S: diamonds, and W: clubs, if spades are trump, Declarer automatically captures all 13 tricks in the North hand. With all other trump choices, 11 tricks are the most that will result. An example sequence that can't be bettered: With diamonds being trump, West puts North in with an opening spade to first collect three spades and then lead a fourth. East discards a club, South does not trump, and West trumps. On the next trick, Declarer gets back in and leads his clubs. The third one is trumped by East. With the balanced distribution, Declarer has no more tricks that can be given away, so 11 tricks result. Declarer not having an entry to South's long diamond prevents Declarer from leading his fourth diamond to allow Defense another two tricks through ruff/sluff as was done with spades. If hearts or clubs are trump, there is a variant where Defense just waits out Declarer's trumps for Defense's second trick. Declarer's 11 tricks equates to a 5 bid, sufficient for game in any trump suit.

S/O 2. Victor Barocas's non-transitive dice problem refers to an example with three dice and two players. Each player selects a die and rolls it. The second player can always select a die to produce an advantage over the first. For a game with N dice, how many faces must a die have, and how do you number them?

Frank Rubin points out that it is not clear if the generalization is to N dice and two players or to N dice and $N-1$ players. Rubin later showed that the latter interpretation yields no solution. Barocas agrees that the generalization is to maintain two players.

Robert Rorschach notes that we should have made all the requirements explicit. For example, if all you want is that whatever die the first player picks, the second can pick one that beats it, the problem becomes trivial. Just use one die like A in the problem statement, one like B, and $N-2$ like C.

The consensus seems to be that we must have "cyclic transitivity," i.e., die 1 beats die 2, die 2 beats die 3, ..., die $N-1$ beats die N, and die N beats die 1.

Anthony Bielecki has described an algorithm that in principle can generate cycles of dice for any N. He sent software in addition to the textual description. Because of space constraints, I have placed both on the Puzzle Corner website, as well as a detailed solution from Greg Muldowney.

Finally, Timothy Chow sent the following solution. It seems almost too good to be true. Any comments for BLTN?

"Here's a way to create $N \geq 3$ non-transitive, three-sided dice using each of the numbers from 1 to $3N$ exactly once. The ith row of this table lists the three numbers on the faces of the ith die:

6	5	4
$3N$	3	2
$3N-1$	$3N-2$	1
$3N-3$	$3N-4$	7
$3N-5$	$3N-6$	8
$3N-7$	$3N-8$	9
$3N-9$	$3N-10$	10
$3N-11$	$3N-12$	11
...		
$N+5$	$N+4$	$N+3$

"Each die beats the die immediately below it, and the last die beats the first die.

"If you want more faces on the dice, then you can just put zeroes on all the other faces."

S/O 3. Dick Miekka and three friends have to cross a narrow footbridge in the dark using only one flashlight, which has 17 minutes of battery power. No more than two people can be on the bridge at one time and anyone on the bridge must be with the flashlight. The four people—A, B, C, and Dick—take respectively one, two, five, and 10 minutes to cross the bridge. How can all get across before the battery runs out?

This problem was popular and well liked. The following solution is from Naomi Markovitz:

A & B	go across	2 minutes
A	returns	1 min (total 3 min)
C & Dick	cross	10 min (total 13 min)
B	returns	2 min (total 15 min)
A & B	cross	2 min (total 17 min)

Diego Puppin informs us that this problem is known as the "U2 band" problem and refers us to https://www.braingle.com/brainteasers/515/u2.html.

Better Late Than Never

M/J 1. I mistakenly printed the wrong hand for 1 no-trump. Robert Virgile, the author of the solution, noted that his actual submission used only three high-card points. Robert Wake also reported that the correct solution has three points. Sorry for the error.

M/J SD. Both Alan Robock and Peter Marmorek noted that I shamefully assumed miles, ignoring kilometers.

Other Responders

Responses have also been received from R. Alladin, M. Branicky, J. Casalegno, W. Chan, S. Chin, J. Freilich, D. Fulton, R. Giovanniello, P. Groot, H. Gross, D. Grunberg, J. Harmon, A. Hirshberg, S. Kanter, D. Karlan, T. Kelly, P. Kramer, B. Kulp, E. Kutin, J. Licini, J. Macro, M. Malak, R. Marks, B. Marshall, D. Mellinger, D. Micheletti, T. Mita, R. Morgan, B. Norris, A. Ornstein, P. Paternoster, R. Ragni, A. Raynus, B. Rhodes, B. Schargel, P. Schottler, J. Sinnett, M. Strauss, M. Subramanian, C. Swift, S. Tamura, T. Tamura, D. Turek, T. Vatcha, B. Wake, D. Worley, and J. Yuan.

Proposer's Solution to Speed Problem

Yes.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 60 Fifth Avenue, Room 316, New York, NY 10011, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.