This issue of *MIT Technology Review* is profiling John Urschel, the MIT mathematics PhD student who is also a member of the Baltimore Ravens (see “The Double Life of John Urschel,” page 12). Way back when I was an MIT math major we would have admired someone like John, who could carry with ease the slate tablets we used for taking notes in class. Naturally, he would have been even more admired for his prodigious mathematical ability. In his spare time from football and grad school, John has written an online puzzle column for *The Players’ Tribune*, and he has provided a sample puzzle for Puzzle Corner (see J/A2 below). I thank him for the contribution and wish him well, except when the Ravens play *my* team. Go, Jets!

As mentioned last issue, we are critically short of bridge/chess/Go/etc. problems.

Problems

J/A 1. Duncan Ewing notes that with the developments in computer Go and chess, there is interest in how many moves are available at one turn. In Go, the maximum number occurs during the first turn of every game. There are then $19 \times 19 = 361$ points on which one can place the initial stone. Duncan asks what is the maximum number possible for chess. Let me refine the question to state that we seek a *reachable* position (i.e., one obtainable via sequence of legal moves from the initial position), from which the number of legal moves is maximal.

J/A 2. John Urschel offers this probability puzzle, which originally appeared in his “Wednesday Morning Math Challenge” in *The Players’ Tribune*:

You have a deck of 52 cards. Aces take the value 1, and face cards take the value 10. You randomly pick a card from the deck, note its value, and return it. You shuffle the deck and then randomly pick another card. Which is more likely, that the sum of the two card values is even or odd?

Now pick four cards instead of two (again randomly, with replacement). What is the probability that the sum is even?

What about 100 cards?

J/A 3. Frank Rubin reports that four mathematicians decide to play a number game. Each one will pick an integer greater than 2, and then they will take turns trying to find relationships among those numbers. The first one says, “There is a unique way in which powers of your numbers add to a 20-digit power of my number.” The second one says, “There is a unique way in which powers of your numbers add to a 22-digit power of my number.” The third one says, “There is a unique way in which powers of your numbers add to a 26-digit power of my number.” The last one says, “There is a unique way in which powers of your numbers add to a 34-digit power of my number.”

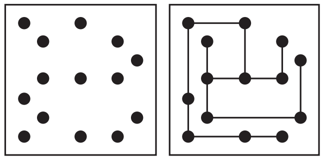
What numbers had they picked?

Speed Department

Duffy O’Craven wants to know the numerically largest Roman numeral that is a normal English word.

Solutions

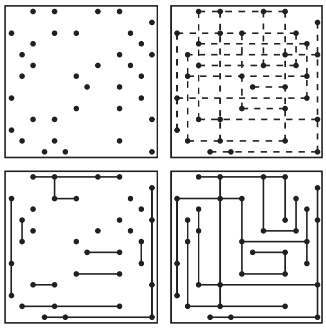
M/A 1.



We begin with Yashi, another puzzle from Frank Rubin’s website sumsumpuzzle.com. As shown in the 7×7 example above, you are given a collection of dots situated on an $n \times n$ integer grid and seek a solution using horizontal and vertical lines to connect all of the dots without any crossings. As illustrated in the solution above, there must be exactly one path connecting any dot to any other. More information, other examples, and useful tips can be found on the website.

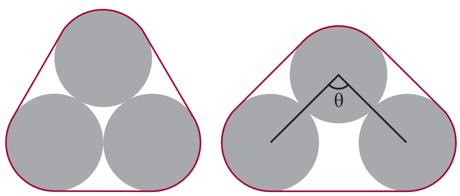
Rubin offered us a 14×14 challenge to which G. Muldowney sent us the following four-step solution.

1. The starting array of points (upper left).
2. All possible connecting line segments (upper right).
3. “Free” line segments, which cross no others (lower left) and therefore must form part of the solution.
4. Complete solution (lower right) determined by ruling out line segment combinations that include crossings.



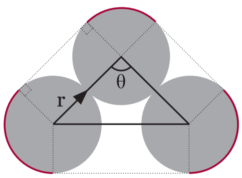
M/A 2. Joe Horton and Marcelino Gorospe report that the following problem was an offshoot of their designing a series of devices for treating brain aneurysms. The first diagram below shows three gray mutually tangent circles of radius r enclosed in a red band. (In the medical device the circles are cross-sections of three thick wires and the band is a thin wire.) What is the length of the band in terms of r?

In the second diagram the circles have moved apart so the bottom two are no longer tangent. To be specific, we show the angle θ (which, by symmetry, was $\pi/3$ before) and ask for the band length in terms of r and θ.



Jim Casalegno went straight to the second, more general situation and then considered the special case θ = 60°. Robert Rorschach notes the following generalization: Take an arbitrary convex planar polygon and draw a circle at each vertex, each with the same radius. Wrap a band around the whole just as in the published problem. The length of the band will be the circumference of one of the circles plus the perimeter of the original. Casalegno's solution follows.

The band around the three circles consists of two types of geometric objects: a straight line segment between circles and a circular arc wrapped around part of each circle. From the figure below, it is clear the straight line segments are identical to the sides of the triangle joining the circle centers. This is because any straight line segment touches a circle at a single point, and the line from that point to the circle's center is perpendicular to the line segment and is of length r, the radius of the circle. Because the two lines perpendicular to a line segment are the same length, the rectangular-looking boxes are truly rectangles, making the inner triangle's sides the same as the band line segments.



The triangle connecting the centers is isosceles, with the equal sides each being of length $2r$. The base is $2(2r)\sin(\theta/2)$, making the total triangle perimeter $4r(1+\sin(\theta/2))$.

The portion of the band that wraps around the circles is shown in the figure in red. The combined length of these three segments is $2\pi r$. To see this, start at the bottom of the left circle where the wrap begins and continue clockwise until reaching the tangent point where a straight line segment begins. The wrapping resumes on the upper circle at the identical angle where the previous wrap left off. Continue the wrap clockwise until reaching the next tangent point, where the wrapping again stops. The wrap around the right-hand circle resumes at the same angle where the previous wrap left off and continues to the identical angle where the wrap began on the first circle, completing a full 360° around a circle of radius r. Combining the circumference of the circle with the straight line segments yields the overall length of the band: $2\pi r + 4r(1+ \sin(./2))$. When the circles are all touching, θ is 60° and band length simplifies to $2\pi r + 6r$.

M/A 3. Eric Schonblom knows a woman who despises pennies (especially the silver Lincoln cents) to such an extent that she never carries any and is normally unwilling to purchase any item for which change must include at least one penny.

Recently this woman emptied some newly minted U.S. coins onto a counter that has glass jars containing candy. Each sort of candy has a different price, a positive integer number of U.S. cents. The coins she emptied onto the counter would permit her to buy two candies from any of the jars, but in each case she would need to supply an additional penny. She also has the right amount of coins to pay for three candies from any jar, but in this case she would need to receive an unwanted penny in change.

In despair she gives up and asks for one candy from each jar, expecting the worst. However, she is pleasantly surprised to find that her coins permit her to pay for the purchase, with no change coming.

How many jars are there, what is the price of the candy in each, and what coins did she have in her purse? The preferred solution is the one with the fewest jars and the fewest coins.

The solution below is from William Lemnios. Terence Sim notes that there are, in addition to this "minimal" solution, 381 others.

There are three conditions that must be met.

1. Two candies from one jar cost one cent more than any combination of coins.
2. Three candies from one jar cost one cent less than any combination of coins.
3. One candy from each jar costs the exact amount for some combination of coins.

The first two conditions are met if the price of a single candy in jar j is $p_j = 3 + 5k_j$ for some $k_j \geq 0$. If there are N jars, the total price of a single candy from each jar is

$$P = 3N + 5\sum_{j=0}^{N-1} k_j.$$

P must be divisible by 5 (since all her coins are); so N must be divisible by 5. The smallest number of jars and coins occurs when $N = 5$ and $k_j = j$. So there are five jars; the price of a single candy in each jar is 3, 8, 13, 18, and 23 cents; and the woman has one nickel, two dimes, one quarter, and one half-dollar.

Other Responders

Responses have also been received from F. Albisu, J. Arsenault, M. Bolotin, M. Brill, P. Cassady, B. Chapp, T. Chase, P. Chin, C. Cox, T. Cox, P. Davis, D. Diamond, R. Giovanniello, J. Grossman, J. Hardis, B. Heflinger, A. Hirshberg, C. Jackson, C. Jacobs, H. Joseph, S. Korb, J. Krupp, R. Kurtz, I. Lai, S. Lerman, Z. Levine, J. Mackro, T. Malony, M. Marinan, N. Markovitz, T. Mattick, D. Mellinger, Z. Mester, D. Micheletti, D. Milliken, T. Mita, R. Morgan, G. Muldowney, E. Nadler, L. Nissim, A. Ornstein, J. Prussing, A. Raynus, B. Rhodes, K. Rosato, P. Schottler, E. Sheldon, E. Signorelli, J. Sinnett, S. Sperry, P. Steven, G. Stith, M. Strauss, C. Swift, D. Weber, S. Yaung, J. Yuan, and R. Zellers.

Proposer's Solution to Speed Problem

MID = 1,499. Can you do better?

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.