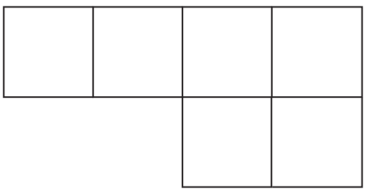
This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 7) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2016 yearly problem is in the "Solutions" section.

Problems

Y2017. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 7 exactly once each, along with the operators +, −, × (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 7 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Another hexominoes problem from Richard Hess and Robert Wainwright. You are to design a connected tile so that five of them cover at least 93 percent of the area of the hexomino below. The tiles must be identical in size and shape and may be turned over so that some of them are mirror images of the others. They must not overlap each other or the border of the hexomino.



J/F 2. Richard Bronowitz offers this radar detection problem. Assume that a radar has a detection threshold requiring at least nine successful pulse returns out of 10 successive pulses. Furthermore, once an object is detected, it remains detected—i.e., there are no lost contacts. The probability that a pulse is successfully detected is *p*, and pulse results are independent. What is the probability of detection given *N* total pulses?

Speed Department

David Dewan wants you to express the following equation as a timely cheerful greeting.

$$Y = \frac{\log_A E + log_A \dfrac{WN + RAY}{H}}{P^2}$$

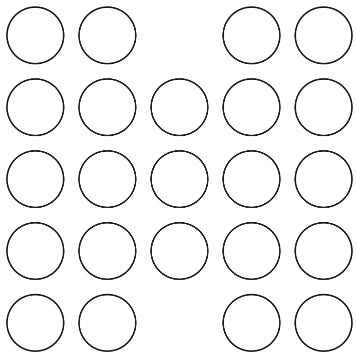
Solutions

Y2016. Burgess Rhodes had fun with this one. His solution for the stated problem follows. He then sent solutions for each year in MIT's first century (1861–1960), its second century, and its third century. Finally, I have placed his report "Counting UNEs" on the Puzzle Corner website (a UNE is an unsimplified numerical expression).

$1 = 261^0$	$20 = 120/6$	$52 = 62-10$
$2 = 2+0^{16}$	$21 = 20+1^6$	$53 = 106/2$
$3 = 2+61^0$	$22 = 10+2\times 6$	$54 = 2^6-10$
$4 = 20-16$	$25 = 20-1+6$	$57 = 60-1-2$
$5 = 60/12$	$26 = 20+1\times 6$	$58 = 60^1-2$
$6 = 10+2-6$	$27 = 20+1+6$	$59 = 60+1-2$
$7 = 21^0+6$	$29 = 60/2-1$	$60 = 60\times 1^2$
$8 = 2\times(10-6)$	$30 = 10/(2/6)$	$61 = 60-1+2$
$9 = 2+0+1+6$	$31 = 60/2+1$	$62 = 2+10\times 6$
$10 = 2\times(0-1+6)$	$32 = (2+0)\times 16$	$63 = 60+1+2$
$11 = 10/2+6$	$35 = 210/6$	$64 = ((2+0)\times 1)^6$
$12 = 6^0\times 12$	$36 = 20+16$	$65 = (1-0)+2^6$
$13 = 20-1-6$	$37 = 1-0+6^2$	$72 = 60+12$
$14 = 20-1\times 6$	$39 = 60-21$	$74 = 10+2^6$
$15 = 20+1-6$	$40 = (6-2)\times 10$	$80 = 160/2$
$16 = 26-10$	$41 = 61-20$	$81 = 20+61$
$17 = 102/6$	$46 = 10+6^2$	$94 = 10^2-6$
$18 = 2+0+16$	$48 = 60-12$	$96 = 102-6$
$19 = 20-1^6$	$49 = (6-0+1)^2$	$100 = 20\times(6-1)$

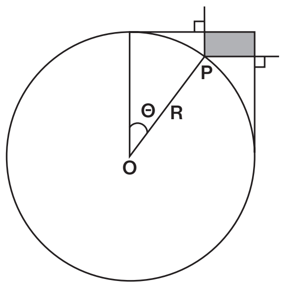
S/O 1. Phillip Davis (and Lady Luck) solved Rocco Giovanniello's "rudimentary H-array" winks problem with (3,3) initially empty. I denote moves as (row, column)-to-(row, column). Davis writes:

"From the symmetry, if there is a solution there is at least one additional as a mirror image of each move. It is also obvious that the last move must be either (3,1)–(3,3) or (3,5)–(3,3). Working backward from the last move seemed as difficult as forward from the first, so I started forward. I got lucky on my second try! Moves #11 and #13 may be interchanged, as may #16 and #18, leading to at least three additional solutions, but life is brief."

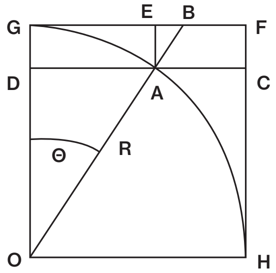


1: (3,1)–(3,3)	8: (3,5)–(3,3)	15: (3,4)–(3,2)
2: (1,1)–(3,1)	9: (1,5)–(3,5)	16: (5,1)–(3,5)
3: (1,2)–(3,2)	10: (4,3)–(2,3)	17: (3,1)–(3,3)
4: (2,4)–(2,2)	11: (2,2)–(2,4)	18: (5,5)–(3,5)
5: (4,3)–(2,3)	12: (5,4)–(3,4)	19: (5,2)–(3,2)
6: (3,1)–(3,3)	13: (3,5)–(3,3)	20: (3,2)–(3,4)
7: (2,3)–(4,3)	14: (1,4)–(3,4)	21: (3,5)–(3,3)

S/O 2. Joseph Horton's diagram illustrated a circle O touching a right angle at point P. Horton wanted to know the area of the shaded region. How about the special case where Θ = 45°?



Zoltan Mester answers both questions.



OA = OG = OH = CD = R
AB = OB – R = R/cosΘ – R = R(1/cosΘ – 1)
OADΔ ≈ ABEΔ
AE = AB cosΘ = R(1/cosΘ – 1)cosΘ = R(1 – cosΘ)
AC = CD – AD = R – AD = R – R sinΘ = R(1 – sinΘ)
Area of ACEF rectangle = AC × AE = $R^2(1 – \sin\Theta)(1 – \cos\Theta)$
When Θ = 45° area is a square
Area= $R^2 (1 – \sqrt{2}/2)^2 = 0.08579\ R^2$
This is the maximum area of such rectangles.

S/O 3. R. P. Mayor's basket problem was not easy, and no one produced a closed-form solution.

The following solution is from William Lemnios, who begins by (mentally) plotting the parabola in a Cartesian coördinate system where x varies from 0 to L and y varies from 0 to A. He defines H as the length of the handle (parabola).

Since

$$dy = (4A/L)(1 - 2x/L)dx$$

we have

$$dy^2 = (4A/L)^1(1 - 2x/L)^2dx^2$$

Since

$$dH^2 = dx^2 + dy^2$$

we get

$$H = 2\int_{x=0}^{x=L/2} dH = 2\int_{x=0}^{x=L/2}\sqrt{1+(4A/L)^2(1-2x/L)^2}\ dx$$

Integrals of this form can be found in several tables of integrals, with the following results:

$$\int\sqrt{au^2 + c}\ du = (u/2)\sqrt{au^2 + c} + (c/2\sqrt{a}) \log (u\sqrt{a} + \sqrt{au^2 + c})$$

For our problem $u = 1 - 2x/L$, $du = -(2/L)\,dx$, $a = (4A/L)^2$, and $c = 1$. When $x = 0$, $u = 1$ and when $x = L/2$, $u = 0$.

The integral becomes

$$H = L\int_0^1 \sqrt{au^2 + c}\ du$$

After integrating the above, we obtain

$$H/L = [\,(u/2)\sqrt{au^2 + c} + (c/2\sqrt{a}) \log (u\sqrt{a} + \sqrt{au^2 + c}\,)\,]_0^1$$

Evaluating at $u = 0$ and $u = 1$ gives

$$H/L = (1/2)\sqrt{1+(4A/L)^2} + [1/8\,(A/L)] \log (4A/L + \sqrt{1+(4A/L)^2})$$

We can iterate the last equation to find a value of A/L such that $H/L = 2$. To five significant figures that value is $A/L = 0.81722$.

Better Late Than Never

S/O speed. Many readers pointed out (politely) that I mis-worded the solution. The maximum value is $e^{1/e}$, which *occurs* at $x = e$. In addition, I should have limited the problem domain to $x > 0$.

David Kessel wrote:

Seeing this in the Speed Department of the recent issue of *MIT Technology Review* reminded me of my own encounter with the problem in about 1949. It occurred in a calculus final for a class run by Gordon Raisbeck, who was otherwise noted for having married one of Norbert Wiener's daughters. I had somehow missed seeing the final problem, and with minutes left before time ran out, I took a guess that the answer had to be more than 2 and perhaps a bit less than 3, so the number e seemed reasonable. To explain why I had omitted the intermediate steps, I claimed that I had a simple proof but that there was not enough space left to write it down.

Raisbeck left this answer unscored but added a "See me" comment. He was, as I recall (with some vagueness since this is almost 70 years later), not entirely amused but decided to give me half credit or $1/e$, whichever was greater. I settled for half.

Other Responders

Responses have also been received from R. Bator, R. Bird, P. Cassady, M. Chartier, E. Collins, B. Daniel, A. Esler, E. Field, R. Ganapathi, M. Gerardi, D. Goldfarb, P. Gottlieb, R. Goutte, A. Hirshberg, H. Hodara, D. Katz, P. Kramer, J. Mackro, J. Marlin, T. Mita, R. Morgan, E. Nelson-Melby, A. Ornstein, T. Palmer, J. Prussing, R. Ragni, B. Rhodes, A. Sezginer, I. Shalom, J. Shapiro, R. Shapiro, P. Sherwood, E. Signorelli, J. Steele, A. Stern, M. Strauss, T. Tamura, M. Thompson, R. Vale, S. Vatcha, M. Weiss, T. Weiss, B. Wells, J. Wrinn, T. Yen, and S. Young.

Proposer's Solution to Speed Problem

$HA^{PPY} = NEW + YEAR$

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.