Having failed to do so because of a space crunch last issue, I must review the ground rules this time. In each issue I present three regular problems, the first of which is normally related to bridge, chess, or some other game, and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns later, one solution is printed for each; I also list other readers who responded.

The solutions to the problems in this issue will appear in the March/April column, which I will need to submit in mid-December. Please try to send your solutions early. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given on the facing page. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that have remained unsolved.

Problems

N/D 1. Another "Texas Hold'em" problem from Richard Morgan. In this poker game a number of players—six, in this case—are each dealt two so-called hole cards, and there are five other "community cards" that all players can use. Thus, each player has access to seven cards (two hole, five community), and each chooses five of them as a hand.

Morgan wants to know the odds that one player's hand contains four nines and another player's hand contains a nine-high diamond straight flush. This was the basis of a promotion at a local casino. That casino required each hand to include both of the player's hole cards. I am asking for the odds both with and without this last requirement.

N/D 2. Two more "THIRDS" puzzles, these from Geoffrey Coram. Use the letters from the third of the alphabet shown to replace the dashes and produce a standard English word.

–L–P––NT (ABCDEFGH)
––––EY (IJKLMNOPQ)
–AL––– (RSTUVWXYZ)
and
–IR–––– (ABCDEFGH)
D–––H––(IJKLMNOPQ)
––L–––E (RSTUVWXYZ)

N/D 3. Our last regular problem is from John Kotelly, who attributes it to his 1954 Boston Latin teacher Frank Gilbert. What is the side length of an equilateral triangle containing an interior point that is respectively three, four, and five units away from the three vertices?

Speed Department

After 120 years Sorab Vatcha adapted this problem from an 1892 Stanford entrance exam. If a, b, and c are in harmonic progression, what is the value of the following?

$$\frac{b+a}{b-a}+\frac{b+c}{b-c}$$

Your editor reminds everyone, including himself, that a sequence is a harmonic progression if the reciprocals form an arithmetic progression.

Solutions

J/A 1. All the solutions I received involved essentially the same distribution for South and West and noted that East's and North's holdings are immaterial.

Mark Bolotin's lucid solution follows.

West has
♠ A Q 10 8 6 4 2
♥ K J 9 7 5 3

South has
♠ K J 9 7 5 3
♥ A Q 10 8 6 4 2

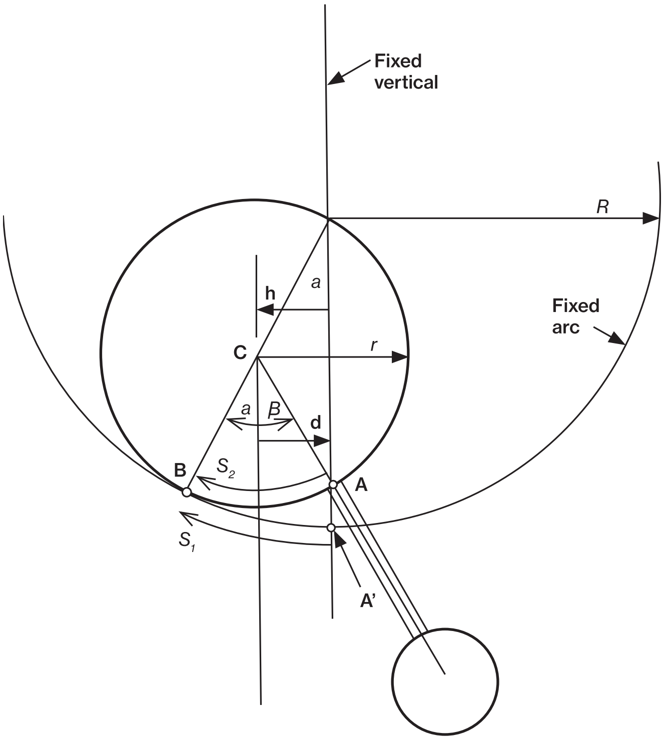
South can make seven hearts with helpful defense. West leads anything except the ace of spades. South wins this as cheaply as possible, saves the heart 2 for trick 13, but plays the other cards in any order. West plays one card lower than South at each opportunity and discards the spade A on the heart 2 at trick 13.

Similarly, South can hold himself to one trick in hearts. West saves the spade 2 until trick 13, but can play his other cards in any order. South plays the next lower card at every opportunity and ruffs the spade 2 with the heart A for his only trick. The North and East hands are irrelevant. Interestingly, there is the same 12-trick potential difference if the trump suit is spades, and a 13-trick difference in no-trump.

J/A 2. I remember that when I first read this problem, I looked for the typo since it was "clearly impossible" as written. However, Herb Helbig referenced http://math.ucr.edu/home/baez/rolling/rolling_3.html and supplied the advice to scroll down to "Twice as big." This website, and those it references, opened my eyes. I advise everyone not familiar with this material to check it out.

The following solution and beautifully drawn diagram are from Timothy Barrows.

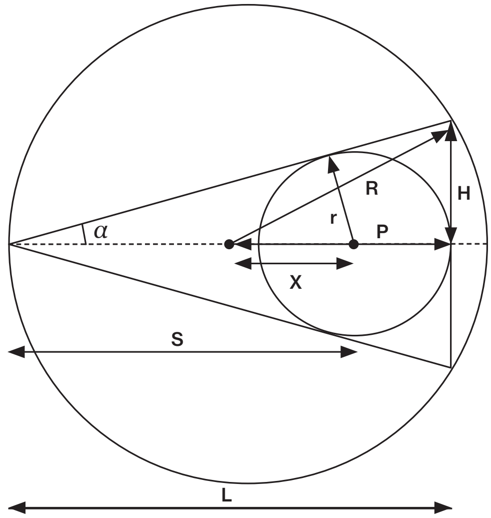
The objective is to find a pendulum whose center of mass moves purely in the vertical direction. The following diagram shows a candidate pendulum consisting of an upper disc, a bar, and a lower disc, all fixed together as one body. The upper disc has radius r and rolls on a fixed arc of radius R, where $R = 2r$. The mass of the lower disc is selected so as to locate the center of mass of the entire pendulum at point A on the circumference of the upper disc. When the pendulum is in the equilibrium (vertical) position, the point of contact is at point A' of the fixed arc, and points A and A' coincide.



When the pendulum is tilted at an angle β, as shown, the point of contact moves to point B. Assuming the rolling motion occurs without slipping, the contact arc length must be the same on both surfaces. We use the symbol s to represent arc length. On the fixed arc, we have $s_1 = Ra = 2ra$. On the circumference of the upper disc, $S_2 = r(a + b)$. Setting $s_1 = s_2$ gives $a = \beta$. The horizontal distance h that center C of the upper disc moves relative to the fixed vertical is $h = r \sin a$. The tilting of the pendulum causes the center of mass at point A to be displaced a horizontal distance d relative to the center C, where $d = r \sin \beta$. Since $a = \beta$, we have $d = h$, meaning that the rightward movement of point A due to tilt is exactly compensated by the leftward movement of point C. Thus the center of mass remains on the fixed vertical for all angles of β, as required.

J/A 3. Several readers pointed out that this is a theorem of Euler. Bill Kleinhans refers us to https://en.wikipedia.org/wiki/Euler%27s_theorem_in_geometry. Jay Mackro shows that Euler's theorem is much easier for isosceles triangles. Ed Nadler notes that Wolfram MathWorld's entry for "circumradius" gives the desired relationship. Timothy Maloney and Charles Wampler each sent detailed solutions that now appear on the Puzzle Corner website (cs.nyu.edu/~gottlieb/tr). The following eight-step solution is from Tom Mattick.

Referring to the diagram below, let the half angle of the triangle vertex be α, and introduce dimensions S, L, H, and P as shown.



1. We see that $S = r/\sin(\alpha)$ and $H = L\tan(\alpha)$ and $L - R = P = \sqrt{R^2 - H^2}$
2. Squaring and rearranging gives $R = (L^2 + H^2)/(2L) = (L/2)(1 + \tan^2(\alpha))$ or $L = 2R\cos^2(\alpha)$
3. Since $X = S - R = L - r - R$ we get $L = X + r + R$
4. (2) and (3) imply $\cos^2(\alpha) = (X + r + R)/(2R)$ or $\sin(\alpha) = \sqrt{1 - (X + r + R)/(2R)}$
5. From (1) we see $r/(x + R) = r/S = \sin(\alpha)$
6. Together (4) and (5) give $r/(x + R) = \sqrt{1 - (X + r + R)/(2R)}$
7. Squaring (6) we obtain a quadratic equation for r: $r^2 + r(X + R)^2/(2R) - (R - x)(X + R)^2/(2R) = 0$
8. Solving the quadratic gives $r = (R^2 - X^2)/(2R)$

Better Late Than Never

J/A SD. Many readers responded. One general comment was that the solution is font dependent. Specific comments were that (for most fonts at least) Q should not be on the list and I, J, G, and T should be added.

Other Responders

Responses have also been received from E. Barzelay, R. Bird, M. Coiley, E. Collins, G. Coram, D. Diamond, W. Goodwin, K. Haruta, S. Kanter, C. Katz, J. Kenton, R. Krawitz, P. Manglis, M. Marcou, H. Nesor, M. Phillips, B. Rhodes, N. Rubin, A. Sherman, D. Sherman, E. Signorelli, B. Snyder, M. Thattai, C. Wampler, M. Weiss, and A. Yen.

Proposer's Solution to Speed Problem

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Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.