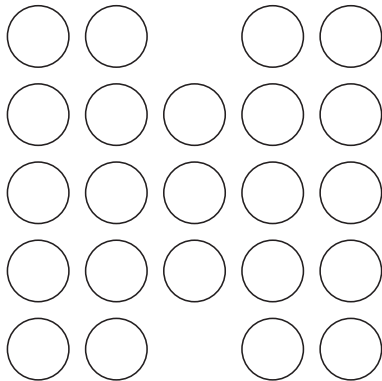


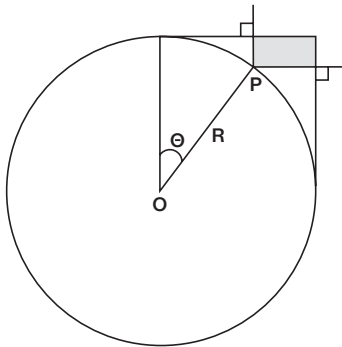
Since this is the first issue of an academic year, it is time for me to review the ground rules. However, we are short of space this issue, so the rules will be given next time.

Problems

S/O 1. Our wink maestro, Rocco Giovanniello, is back with the “rudimentary H-array” depicted below, containing 23 spaces. The games begins with central location (3,3) empty and the other 22 positions containing a wink. Each move is a checkers-like (horizontal or vertical) jump, removing the jumped-over wink. The goal is to reduce the board to just one wink, which (in a pleasing symmetry) now occupies the center space, the one location that was initially empty.



S/O 2. The diagram below, from Joseph Horton, illustrates a circle O touching a right angle at point P. Horton wants to know the area of the shaded region. How about the special case where $\theta = 45^\circ$?



S/O 3. R. P. Mayor wants to design a basket so that the length of the handle is twice the span. If the handle is represented by the parabola

$$y = \frac{4A}{L} (x) \left(1 - \frac{x}{L}\right)$$

where A is the height and L is the span, what is the ratio A/L so that the length of the parabola is equal to twice the span?

Speed Department

Joe Horton wants to know the maximum value of $x^{1/x}$.

Solutions

M/J 1. We begin with the following solution from William Popendorf.

It is obvious that if the bidder holds all 13 trump cards, a grand slam is inevitable. In order to guarantee a small slam (12 tricks), the hand can hold one less trump. Thus, the question becomes: what is the highest-count trump card that can be missing and still guarantee taking 12 tricks?

Let us start with the hands that are missing the highest-point card(s) and work our way down.

The missing card cannot be the ace of trump. If the missing card is the king of trump, the outcome is uncertain. If the opening lead (from the bidder’s left) is to an ace held by the right-hand opponent (RHO) followed by the lead of a second ace and the LHO is void and holds the trump king, then the bidder is finessed.

If the missing card is the queen of trump (i.e., the bidder has eight points), the outcome of a small slam is guaranteed. As above, if the opening lead (from the LHO) is won by an ace who leads back with a second ace, then the bidder can play and win with the ace of trump, to be followed by leading the king of trump, on which the opponent’s queen must be played.

The minimum points needed to guarantee winning 11 tricks is six; the bidder is missing the ace of trump. The opponents win the non-trump trick and their trump ace, but the bidder wins all the rest.

The proposer notes that five points (missing the KQ of trump) is sufficient for 10 tricks. If the LHO has the KQ, and the opponents attempt a trump promotion; the bidder discards the side cards, then permits herself to be overruffed. Now the opponents lack the communication to lead through the bidder a second time.

Scott Nason notes that three points (missing the AK of trump) suffices for nine or eight tricks and one point (missing the AKQ of trump) is sufficient for seven.

M/J 2. Terence Sim sent us a detailed solution, complete with very helpful diagrams. However, it is too long to appear here, so I have posted it on the Puzzle Corner website. Ted Mita sent us the following more compact solution.

A’s hat is 6, and B’s hat is 7. If B sees a 1 on A’s hat, he knows his own number is 2. A’s first comment (that there is no way B can know his own number) precludes this.

If B sees a prime number P on A’s hat, he knows his own number is $P+1$. A’s comment that there is no way B can know his own number precludes a situation where B’s number can be expressed as a prime plus 1, because then A might have a prime number on his hat. When B’s number can be expressed as a prime plus 1, we will call that a “prime-plus-one situation.”

Thus, B’s seeing 2 or 3 is precluded.

If B sees 4, he knows he has either 4 or 5 (2 + 2 or 1 + 4). But B’s having a 4 would be a prime-plus-one situation, which is precluded, leaving him knowing he had a 5. But B’s first comment (that he does not know his number) precludes that. Thus B’s seeing a 4 is precluded.

B's seeing a 5 is a prime-plus-one situation, which is precluded. If B sees a 6, he knows he has either 5 or 7 (2 + 3 or 1 + 6). A's first comment does not help him decide, so B makes his first comment, that he doesn't know his number. But upon hearing A's second comment, that A doesn't know her own number, B reasons as follows:

"If I had 5, then A would reason as follows: 'I must have either 4 or 6 (1 × 4 or 2 × 3). But if I had a 4, B would know he had a 4 or 5 (2 + 2 or 1 + 4), and my first comment would let him eliminate 4 and conclude he had a 5. So his saying he doesn't know his own number would be illogical. Thus, I have a 6.'"

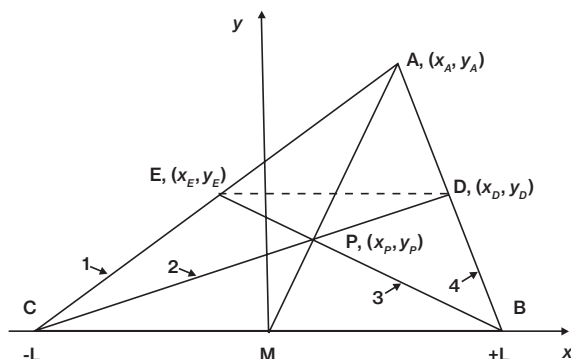
B continues thinking ...

"But A says she doesn't know her own number. This is illogical. Therefore, I have a 7."

B declares that he knows his own number, 7.

Thus, x and y being 1 and 6 leads to A having 6 and B having 7, and the dialogue we are given is logical. Showing that this is the only solution is a whole other ball of wax, I surmise.

M/J 3. Apparently the slick way to prove this assertion is to appeal to Ceva's theorem, which is in Wikipedia but is beyond my knowledge of geometry. Another method used is to extend AM beyond BC and find a parallelogram. Instead of these, I am printing Tim Barrows's solution, which is in the spirit of Cartesian geometry and thus more familiar (at least to me).



The problem consists of proving that triangle ADP has the same area as triangle AEP. We first set up a coordinate system whose origin is at the midpoint M of the baseline of triangle ABC. Let L be the distance from M to B. Then points B and C have coordinates (L, 0) and (-L, 0), respectively. The proof begins by proving that the dashed line ED is parallel to the baseline (the x axis). Lines CA, CP, BP, and BA are numbered 1 through 4 as shown. Each of these lines can be described by an equation of the form $y = m_i x + b_i$, where i is one of these numbers. It is straightforward to solve for the slopes and intercepts:

$$\begin{aligned} m_1 &= \frac{y_A}{x_A + L} & b_1 &= \frac{y_A L}{x_A + L} \\ m_2 &= \frac{y_P}{x_P + L} & b_2 &= \frac{y_P L}{x_P + L} \\ m_3 &= \frac{y_P}{x_P - L} & b_3 &= \frac{-y_P L}{x_P - L} \end{aligned}$$

$$m_4 = \frac{y_A}{x_A - L} \quad b_4 = \frac{-y_A L}{x_A - L}$$

Given two lines $y = m_i x + b_i$ and $y = m_j x + b_j$, the y coordinate of the point of intersection is

$$y = \frac{m_i b_j - m_j b_i}{m_i - m_j}$$

Point D is at the intersection of lines 2 and 4. Thus we substitute $i = 2, j = 4$ into this to get

$$y_D = \frac{-2 y_A y_P L}{y_P (x_A - L) - y_A (x_P + L)}$$

For point E, we substitute $i = 1, j = 3$ to get

$$y_E = \frac{-2 y_A y_P L}{y_A (x_P - L) - y_P (x_A + L)}$$

Since point P lies on line MA, we have $y_P = y_A x_P / x_A$. Substituting this into the two preceding expressions, we obtain

$$y_D = y_E$$

This proves that the line ED is parallel to the baseline.

The remainder of the proof uses the formula Area = $\frac{1}{2}$ base × height. Letting the symbol Δ mean "the area of the triangle" rather than just "triangle," we immediately see that $\Delta ACM = \Delta ABM$, since the base of each is the length L, and $\Delta PCM = \Delta PBM$ for the same reason. Subtracting these areas,

$$\Delta APC = \Delta APB \quad (1)$$

Since we have just shown that $y_D = y_E$, $\Delta CEB = \Delta CDB$. Subtracting ΔCPB from both of these,

$$\Delta EPC = \Delta DPB \quad (2)$$

Subtracting (2) from (1), $\Delta AEP = \Delta ADP$. *Quod erat demonstrandum.*

Other Responders

Responses have also been received from S. Berkenblit, R. Bird, J. Chandler, D. Dougherty, G. Fischer, J. Francoeur, J. Hardis, H. Hodara, J. Lane, C. Lawes, J. Mackro, R. Menikoff, Z. Mester, D. Micheletti, T. Mita, E. Nadler, A. Ornstein, K. Rosato, E. Ruys, S. Shapiro, S. Silverstein, J. Sinnett, K. Wise, A. Yen, and G. von Ruden.

Proposer's Solution to Speed Problem

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Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr. Share your favorite puzzle from the last 50 years at PuzzleCorner@technologyreview.com.