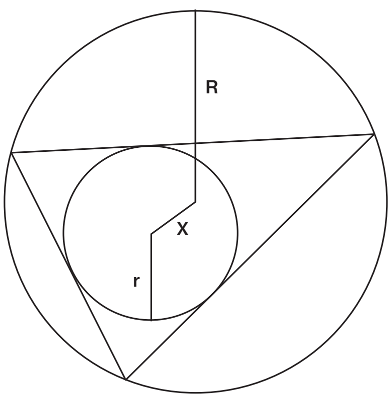
My broken-ankle experience has taught me several things. First, I had incorrectly reported my injury as a broken leg since I broke the fibula, which is a leg bone. But it turns out fracturing the bottom part of the fibula is considered a broken ankle. More important, I now have a significantly greater understanding of the justification for the Americans with Disabilities Act. Even now, 2.5 months later, I avoid the very crowded Lexington Avenue express train during rush hour, opting instead for the (shudder) local. I notice the lack of subway elevators, and when I first returned to NYU I needed a cane or a helping hand when traversing an old, steep ramp. Adding elevators is expensive, but lengthening the ramp would not have been, had anyone thought of it during construction.

## Problems

**J/A 1.** Larry Kells wonders if you can find a suit contract where the difference between the maximum number of tricks for the declarer is 12 more than the minimum, assuming all players are coöperating to achieve this goal. If it is not possible to obtain a difference of 12 tricks, what is the largest value possible?

**J/A 2.** Herb Helbig wants you to construct a single-piece, rigid, unjointed pendulum whose center of mass moves in a strictly vertical direction as the pendulum oscillates.

**J/A 3.** A geometry problem from Gary Rosenthal, who has a small circle of radius $r$ and a big circle of radius $R$. Their centers are a distance $X$ apart. What is the relationship between $r$, $R$, and $X$ so that the big circle circumscribes a triangle that has the small circle inscribed?




## Speed Department

Richard Cartwright wonders: what is special about the letters A, E, F, H, K, Q, X, and Y?

## Solutions

**M/A 1.** Donna Levin and Denis Loring created a crossword puzzle they thought might amuse Puzzle Corner readers.

This problem was popular and, as far as I can tell, has a unique solution exemplified by the following response from Ian Lai.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | M | A | G | E | | S | A | W | S | | N | A | M | E |
| N | E | V | E | R | | P | L | O | W | | E | L | A | N |
| S | T | I | N | E | | A | S | E | A | | W | I | N | G |
| | A | V | O | C | A | D | O | S | N | U | M | B | E | R |
| | | | M | T | G | E | | | D | R | O | I | D | S |
| C | H | E | E | S | E | | A | M | I | G | O | | | |
| H | E | M | S | | | O | L | I | V | E | N | U | L | L |
| U | M | A | | C | O | S | I | N | E | S | | P | A | Y |
| G | O | G | O | L | P | L | E | X | | | T | O | K | E |
| | | | C | A | P | O | N | | A | M | I | N | E | S |
| D | E | L | A | N | O | | | I | B | E | T | | | |
| P | L | A | N | K | S | C | O | N | S | T | A | N | T | |
| L | I | M | A | | I | O | T | A | | E | N | E | R | O |
| U | S | E | D | | T | W | I | N | | O | I | L | U | P |
| S | E | R | A | | E | L | S | E | | R | A | L | E | S |

Stephen Richmond notes that the clue for 39A should have been $10^{10\text{\^{}}100}$. Aaron Hirshberg notes that the clue "prepare to compete for the Mr. Universe title" recently appeared in the *New York Times* Sunday crossword.

Their answer was "oilsup"; ours is "oilup."

**M/A 2.** Brian Cook likes to think big. He wished to merge $N$ companies into one giant company and asks: If only one merger can be performed at a time and only pairwise mergers are allowed ($2 \rightarrow 1$), how many distinct ways are there to form the giant company? For example, if $N = 3$, then there are only three possible orders in which to merge them: ((12)3), ((13)2), and (1(23)). For extra credit, try removing the restriction to pairwise mergers.

Michael Branicky defines $m(N)$ to be the number of ways to merge $N$ companies if only one merger is allowed at a time and only pairwise mergers are permitted. He then proceeds as follows:

Let

$$\binom{n}{m} = \frac{n!}{(n-m)!m!}$$

be the combinations of $n$ things taken $m$ at a time.

It is then easy to see that

$$m(N)=\binom{N}{2}\times m(N\text{-}1)$$

The reason is that one can choose a pair of companies to merge (without loss of generality named "1" and "2"), which then become a single new company (named, say, "1 and 2"). Now you are left with the same problem with $N-1$ companies remaining. Note that the best cases are $m(2) = 1$

and

$$m(3)=\binom{3}{2}*1=3$$

The solution to the recursion can be shown to be

$$m(N)=\frac{N!(N\text{-}1)!}{2^{N\text{-}1}}$$

This may be obtained by expanding the recursion as

$$m(N)=\prod_{i=2}^{N}\binom{i}{2}$$

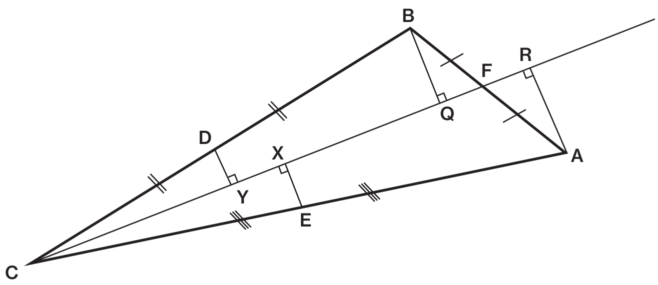
and then expanding and simplifying:

$$\prod_{i=2}^{N}\binom{i}{2}=\prod_{i=2}^{N}\frac{i(i\text{-}1)}{2}=\frac{N!(N\text{-}1)!}{2^{N\text{-}1}}$$

Jerrold Grossman and Burgess Rhodes gave solutions to the extra credit problem; their solutions are on the Puzzle Corner website.

**M/A 3.** Joseph Horton has a triangle ABC with D, E, and F the midpoints of the sides opposite A, B, C respectively. He draws DY and EX perpendicular to CF and wants you to prove DY = EX. The following solution is from Ron Kushkuley.

Extend $\overline{CF}$ and draw altitudes from $B$ and $A$ to $\overrightarrow{CF}$, forming $\overline{BQ}$ and $\overline{AR}$ as shown in the figure below.




From this we can see that triangle $CDY$ is similar to triangle $CBQ$ by AA. Since $D$ is the midpoint of $\overline{CB}$, the side ratio of these triangles is 1:2. resulting in

$$BQ=2(DY). \qquad (1)$$

Similarly, triangle CXE is similar to triangle CRA with side ratio 1:2. Hence,

$$AR=2(EX). \qquad (2)$$

We can easily prove that triangles $BQF$ and $ARF$ are congruent. Since F is the midpoint created by the median $\overline{CF}$, $\overline{BF}=\overline{AF}$. Angles $QFB$ and $RFA$ are equal because they are vertical angles. Also, angles $BQF$ and $FRA$ are both 90° angles. As a result angles $QBF$ and $RAF$ are equal too. By ASA (angle-side-angle), this proves that triangles $BQF$ and $ARF$ are congruent, and

$$\overline{BQ}=\overline{AR}. \qquad (3)$$

Using (3), we equate (1) and (2) to give the desired result:

$$\overline{EX}=\overline{DY}$$

## Better Late Than Never

**2015 J/A 3.** Barry Nalebuff noticed that the signs should alternate in the final formula.

**2015 S/O 2.** Farrel Powsner sent an alternate solution, together with some history on the problem.

## Other Responders

Responses have also been received from S. Berkenblit, R. Bird, M. Branicky, P. Cassady, I. Gershkoff, R. Giovanniello, M. Gordy, Y. Hinuma, A. Hirshberger, A. Kunin, P. Lawes, W. Lemnios, V. Luchangco, J. Mackro, D., J., and M. Marinan, T. Mita, B. Nalebuff, A. Ornstein, P. Paternoster, B. Rhodes, E. Sard, J. Schonblom, P. Schottler, M. Seidel, H. Shane, A. Shuchat, E. Signorelli, P. Silverberg, J. Sinnet, S. Sycuro, A. Wechsung, A. Whitney, and D. Worley.

## Proposer's Solution to Speed Problem

They cannot be hand-printed without lifting the pen or retracing part of the letter.

**Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr. Share your favorite puzzle from the last 50 years at PuzzleCorner@technologyreview.com.**