In late February we had a cold day or two followed by a warm rain. The result, naturally, was ice. My beautiful wife, Alice, told me not to walk to our mailbox, but in the finest tradition of American husbands, I knew better. I hope the cast comes off my broken leg later this week.

I received a lovely letter sent by George O. Peters from Israel. Dr. Peters graduated from MIT two years before I was born! Humbled again.

It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a large queue of regular and speed problems and a comfortable supply of bridge problems.

Problems

M/J 1. A many-part bridge problem from Larry Kells: What is the fewest points you need (in one hand) to guarantee making a small slam? What about 11 tricks? Ten? Nine? Eight? Seven?

M/J 2. Another "logical hat" problem from Dick Hess. Two logicians, A and B, are each wearing a hat with a number affixed. The product $x \times y$ is written on A's hat and the sum $x + y$ on B's, for not necessarily distinct positive integers x and y. Each logician sees the other's hat but not his own. Each is error-free in reasoning and knows the situation. They speak in turn.

A: There is no way you can know the number on your hat.

B: I don't know my number.

A: I don't know my number.

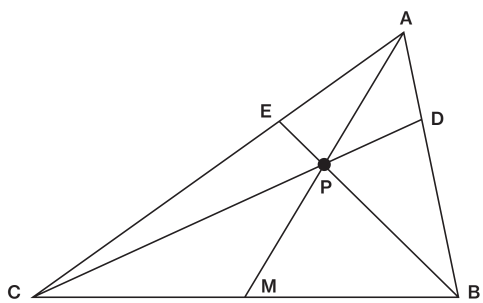
B: I now know my number.

What numbers are on A's and B's hats?

M/J 3. We end this section with a geometry problem from Joseph Horton.

In triangle ABC, M is the midpoint of BC. Draw AM and choose any point P on it. Now extend BP to intersect AC at E and CP to intersect AB at D.

Prove that the areas of triangles ADP and AEP are equal.



Speed Department

Duffy and Mandy O'Craven wonder: What will be the human population of Earth next time it is a Fibonacci number?

Solutions

J/F 1. A poker-inspired combinatorial problem from Richard Morgen, based on a recent game of Texas Hold 'em in which he received a pair of pocket aces and was later surprised to discover that another player did as well. He wonders what the odds are of this happening. Specifically, 10 players each receive two cards from an ordinary, well-shuffled deck of 52. What are the odds of two players each receiving two aces?

Bruce Heflinger writes that he has seen the column since its inception, which occurred his freshman year at MIT. To date he has not responded. Bruce has now crossed that item off his bucket list with the following solution.

As the first 20 cards get dealt, ignore suits and look for a repeated pattern of aces and non-aces between the first 10 cards and the second 10 cards. If, for example, the third player and the sixth player are each to be dealt a pair of aces, abbreviating A for an ace and N for a non-ace, the required sequence of cards is NNANNANNNN, NNANNANNNN.

Let

$$C(n, p) = \frac{n!}{(n-p)! \times p!}$$

Then, ignoring suits, in the entire shuffled deck there are $C(52, 4) = 52!/(4!48!)$ positional arrangements of four aces. In the first 20 cards dealt, there are $C(20, 4) = 20!/(4!16!)$ positional arrangements of four aces. In the first 10 cards dealt, there are $C(10, 2) = 10!/(2!8!)$ positional arrangements of two aces.

Consider a deal in which four aces lie within the first 20 cards dealt. In order for the same two players to receive a second ace, the second 10 cards' pattern is constrained to repeat that of the first 10 cards. Hence there are exactly $C(10, 2) = 45$ positional arrangements of the first 20 cards in which two players each receive two As while the other eight players each receive two Ns.

The probability that two players each receive two hole-card aces is the product of the probability that four aces lie within the first 20 cards dealt with the second probability that the arrangement of those four aces is one of the 45 repeated sequences containing two As and eight Ns.

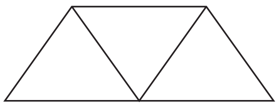
The combined probability is

$$\frac{C(20,4)}{C(52,4)} \times \frac{C(10,2)}{C(20,4)} = \frac{C(10,2)}{C(52,4)} = \frac{45 \times 4 \times 3 \times 2 \times 1}{52 \times 51 \times 50 \times 49} = \frac{1{,}080}{6{,}497{,}400}$$

Thus, the odds are roughly 6,000 to 1 against.

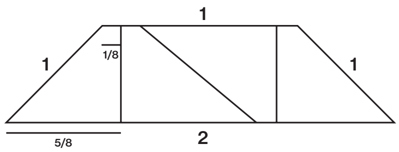
J/F 2. Next we have three more problems from Bob Wainwright's "10 tough tiling tasks" series sent to us by Richard Hess. You are to dissect the equilateral trapezoid composed of three abutting equilateral triangles as shown below into four similar pieces—that is, parts of the same shape. Each part must be connected and must have a finite number of sides.

For the first task, all four parts have the same size. For the second task, three are of one size and the fourth a different size. For the third task, two parts are of one size and two of another size.



The following solution is from Jeffrey Schenkel.

Task one, with all four parts having the same size, can be seen by inspection to be four right trapezoids with base lengths of 1/8 and 5/8, as shown in the following diagram. Note that the height of the original figure is easily seen to be $\sqrt{3}/2$



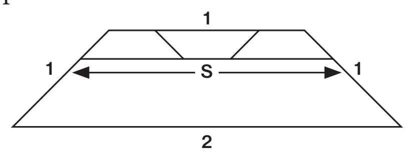
Note that

$$1/8 + 1/8 + 1/8 + 5/8 = 1$$

and

$$5/8 + 5/8 + 1/8 + 5/8 = 2$$

The second task, with three parts one size and the fourth a second, is the most involved, mathematically. As we see below, it utilizes three small isosceles trapezoids atop a single large isosceles trapezoid.



Let α equal the ratio of the smaller trapezoids to the larger and let S equal the length of the top of the larger trapezoid. At the top of the figure, two small trapezoids' shorter bases plus one small trapezoid's longer base equals 1, or $2\alpha S + 2\alpha = 1$. Hence $\alpha = 1/(2S + 2)$. We then observe that S is equal to two small trapezoids' longer bases plus one small trapezoid's shorter base, or $2\alpha 2 + \alpha S = S$ or $S = (4\alpha)/(1 - \alpha)$.

Combining these two results gives

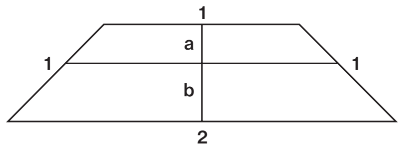
$$\alpha = \frac{1}{\frac{8\alpha}{1-\alpha} + 2}$$

$$6\alpha^2 + 3\alpha - 1 = 0$$

$$\alpha = \frac{\sqrt{33} - 3}{12}$$

Substituting back gives the value for S and the lengths of the smaller bases: approximately 0.271 and 0.457.

The third task, with two parts of one size and two of another, utilizes two pairs of right trapezoids, as shown below.



To satisfy the similar-shape requirement, we note that the ratio of the bases of the smaller and larger trapezoids must be identical. This requires the common base length to be the geometric mean between the smaller base of the smaller trapezoid and the larger base of the larger trapezoid. The solution is $\sqrt{2}/2$.

The heights of the trapezoids (shown as a and b in the diagram) must be in the same ratio and sum to $\sqrt{3}/2$. Some algebra yields

$$a = \frac{\sqrt{3}}{2 + 2\sqrt{2}} \qquad b = \frac{\sqrt{3}}{2 + \sqrt{2}}$$

Better Late Than Never

Y2015. John Chandler, Ermanno Signorelli, and Tom Harriman correct 36 to read $(5 + 1 + 0)^2$ and note that $95 = (20 - 1) \times 5$ is in the preferred order.

S/O 2. Stephen Shalom and Eugene Sard note that the formula for a^2 should end in a superscript, not a plain 2.

Other Responders

Responses have also been received from R. Bird, W. Blank, M. Chartier, J. Coram, J. d'Almeida, R. Ethier, E. Foster, I. Gershkoff, R. Giovanniello, D. Goldfarb, M. Goldring, M. Gordy, J. Grossman, K. Hanf, J. Harmse, P. Kramer, V. Luchangco, T. Maloney, T. Mattick, T. Mita, R. Morgen, O. Ornstein, P. Paternoster, M. Piazza, J. Prussing, K. Rosato, A. Sezginer, I. Shalom, A. Shuchat, D. Simen, A. Soncrant, A. Stern, M. Strauss, D. Turek, and S. Ulens.

Proposer's Solution to Speed Problem

7,778,742,049, unless there is a very dire future, in which case it's 4,807,526,976.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.