Happy birthday/anniversary. This month (August) I, like many other members of the class of ’67, celebrated my 70th birthday. Last month I finished my 35th year at NYU, and this issue completed the 50th year of Puzzle Corner.

On a more youthful note, our two-year-old grandson, Hunter, was here for the weekend together with our two boys, David and Michael, and David’s wife, Mari. Hunter is a joy to be with, and a good time was had by all. My beautiful wife, Alice, and her mother, Eva, were beaming all weekend.

Problems

N/D 1. John Astolfi wants you to find a valid chess position where a legal move by White gives a double check to Black, but the White piece that moves does not deliver a check.

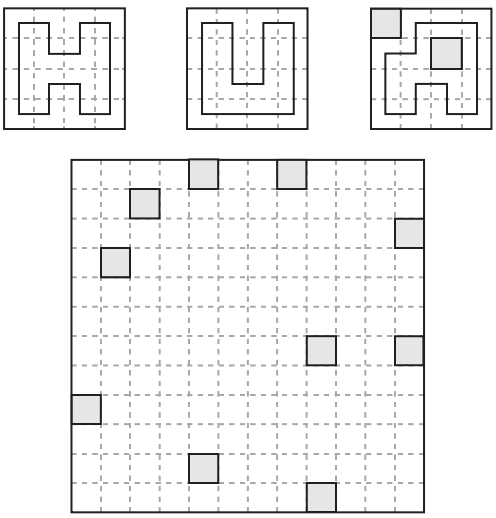
N/D 2. Donald Aucamp notes that hero Jack Reacher in Lee Child’s *A Wanted Man* mulls over (but does not solve) the following problem (paraphrased).

Let $f(y)$ be the reduction of the nonnegative integer y, which is formed by adding all the digits and repeating the process until only a single digit remains. For example, 948 -> 9 + 4 + 8 = 21 -> 2 + 1 = 3, so $f(948) = 3$.

Now let y be the sum of three consecutive nonnegative integers $n - 2$, $n - 1$, and n. Show that $f(y) = 6$ if and only if n is divisible by 3.

N/D 3. Nob Yoshigahara attributes the following problem to Kotani (who presumably spends a lot of time showing houses). In the first two figures, each of the 16 rooms is visited by a “grand tour” exactly once. In the third figure, two rooms are “closed” and only one such grand tour is possible.

The large figure shows a 144-room house with 10 closed rooms. Find the unique grand tour.



Speed Department

Tom McNelly asks: in what number base is 14 squared equal to 176, and why would it be a good choice for government accountants?

Solutions

J/A 1. Jorgen Harmse wants South to make seven spades when West leads the king of hearts.

North:
♠ A
♥ J 10 8 7
♦ A K Q
♣ A 10 9 4 2

West:
♠
♥ A K Q 9 3 2
♦ 7 5
♣ J 7 6 5 3

East:
♠ J 9 7 6
♥ 6 5 4
♦ J 8 4 3
♣ K 8

South:
♠ K Q 10 8 5 4 3 2
♥
♦ 10 9 6 2
♣ Q

Peter Sugar begins by having South cash eight winners as follows.

1. Ruff low in the hand.
2. Lead the club queen to dummy’s ace.
3. Lead a club and South ruffs low.
4. Lead a diamond to dummy’s ace.
5. Leads a heart and South ruffs low.
6. Leads a diamond to dummy’s king.
7. Leads a heart and South ruffs low.
8. Leads a diamond to dummy’s queen.

At this point the situation is (West can differ)

North:
♠ A
♥ J
♦
♣ 10 9 4

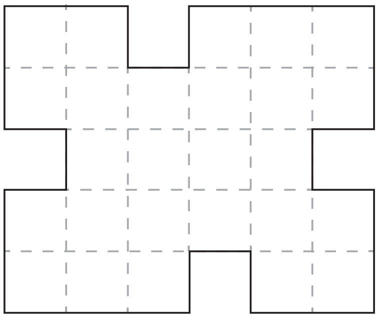
West:
♠
♥ A
♦
♣ J x x x

East:
♠ J x x x
♥
♦ J
♣

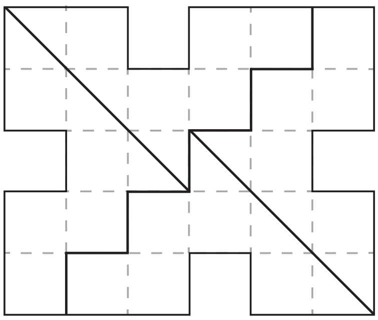
South:
♠ K Q 10 8
♥
♦ 10
♣

Now we lead the heart jack. If East ruffs, South over-ruffs and then leads the diamond 10 and ruffs with the spade ace—East has to play his last diamond. Then dummy plays a club, and whichever spade East plays, his jack is caught. If East at trick 9 plays his diamond jack instead, South ruffs and then leads his diamond 10 to dummy's ace of spades. East has to throw a low spade and the rest is the same: East will ruff dummy's club, South will over-ruff, and again East's jack is caught.

J/A 2. Our next problem, from "Golomb's Gambits" by Solomon Golomb, is one of a series entitled "Figures Drawn and Quartered." You are to dissect the following geometric shape into four congruent pieces.



Ken Haruta found the following solution.



J/A 3. Ermanno Signorelli asks us to suppose that he works at a firm with 4,000 employees and that HR has decided to drug-test 1,000 employees every three months. To be "fair," HR will select the subjects randomly, regardless of whether they have been tested before. What is the probability that at least one employee will not be tested after 10 years? What is the probability that Ermanno himself will not be tested? Assume that no employees either join or leave the company.

The following solution is from Jerrold Grossman. At first I thought his response, which I found clearly correct, differed from everyone else's, which I at first thought were only approximations. However, all is well, as explained below.

Let n be the number of employees (4,000) and k the number tested. Grossman notes that the number of k-person cohorts chosen from all n employees is $\binom{n}{k}$; whereas the number of k-person cohorts excluding Ermanno is $\binom{n-1}{k}$. So the chances that Ermanno was not tested (for just one test) is

$$\binom{n-1}{k} / \binom{n}{k} = \binom{3{,}999}{1{,}000} / \binom{4{,}000}{1{,}000}$$

Nearly everyone else said if 3/4 are not tested, Ermanno's chance of not being tested is 3/4. Fortunately, before I could figure out how to spell "approximation," I realized that the two are equal and saved myself considerable embarrassment.

Returning to Grossman's solution, for t (40) independent trials, the probability that Ermanno will not be selected is

$$e(1) = \binom{n-1}{k}^t / \binom{n}{k}^t$$

This is just a hair over 1/1,000 of 1 percent. (We can't just multiply this by 4,000 to get the answer to the main question, because that overcounts the cases in which more than one person was not tested.)

Similarly, the probability that Ermanno and his boss will not be tested is

$$e(2) = \binom{n-2}{k}^t / \binom{n}{k}^t$$

The probability that at least one employee will not be tested is given by applying the Inclusion-Exclusion Principle:

$$\binom{n}{1}e(1) - \binom{n}{2}e(2) - \binom{n}{3}e(3) - \binom{n}{4}e(4) + \ldots.$$

I don't know a closed form for this, but the terms quickly peter out and the sum is approximately 0.03943, or a little less than 4 percent.

Other Responders

Responses have also been received from M. Azizoglu, R. Bair, R. Bator, G. Biek, S. Blondin, M. Branicky, L. Fattal, J. Feil, G. Fischer, E. Friedman, S. Gordon, M. Gordy, M. Hellinghausen, A. Hirshberg, P. Horvitz, S. Kanter, T. Keane, J. Kolodkin, S. Korb, L. Lerman, N. Markovitz, H. Marks, R. Morgen, B. Nalebuff, S. Nason, E. Nelson-Melby, A. Ornstein, P. Paternoster, G. Perry, J. Prigoff, M. Schatz, I. Shalom, L. Somerville, J. Steele, M. Strauss, W. Sun, and R. Utz.

Proposer's Solution to Speed Problem

Base -10, which has no need for red ink.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.