L et me review the ground rules. In each issue I present three regular problems, the first of which is normally related to bridge, chess, or some other game, and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns later, one solution is printed for each; I also list other readers who responded.

The solutions to the problems in this issue will appear in the January/February column, which I will need to submit in mid October. Please try to send your solutions early. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given on the facing page. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that have remained unsolved.

Problems

S/O 1. Sorab Vatcha takes us into new territory by offering a Scrabble problem. He asks. In the first turn of a new game of Scrabble, what word, when optimally placed on the empty board, would give the highest possible score? What is the score and what is the optimal placement. All the rules of Scrabble apply.

Vatcha requires that the word be in the current "official Scrabble Players' Dictionary". I am refining that requirement to refer specifically to http://www.hasbro.com/scrabble-2/en_US/search.cfm#dictionary.

S/O 2. As an incoming MIT freshman Burgess Rhodes was challenged by an upperclassman with a special case of the following geometry problem. (By comparison, I was challenged to ping pong games.)

Point P inside an equilateral triangle is at a distance a, b, and c from vertices A, B, and C respectively. If $\{a, b, c\}$ is a Pythagorean triangle (i.e., $a^2 + b^2 = c^2$), what is the side-length s of the triangle.
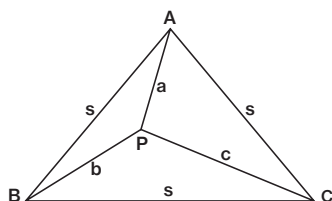
S/O 3. Nob Yoshigahara wants you to place all the L shaped pieces into the rectangular region. You may rotate the L's but may not turn them over. Note that one L is different from the others.
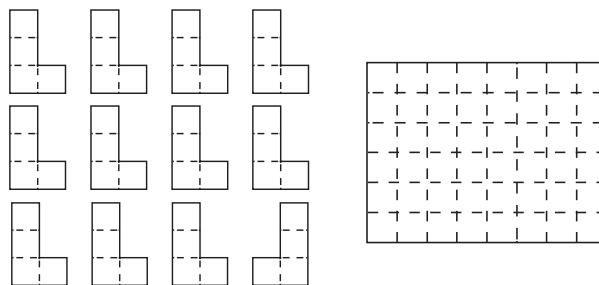
Speed Department

Joseph Horton wants you to find x such that $x \char`^ x \char`^ x \char`^ x \char`^ \ldots = 2$, where $\char`^$ indicates exponentiation and $\ldots$ indicates that there are an infinite number of exponentiations. Don't forget that exponentiation is right associative.

Solutions

M/J 1. Richard Lipes sent us the following explanation why the answers are "No. Yes."

Part 1: A four way tie is not possible. Let n be the number of points each player has at the finish and let d be the number of deals. The solution must satisfy:

$$4 \times n = 26 \times d, \text{ where } n \geq 100$$

The smallest $n \geq 100$ divisible by 26 is $n = 104$.

Before the last deal, each player had to have a score under 100. This means each player had to gain at least 5 points on the last deal (104 - 5 = 99). But one player had to have at least 13 points, so the sum of the points of the other three players is at most 13, which is less than the 15 points these three players had to gain in the last deal. Consequently, no four way tie is possible.

Part 2: A three way tie is possible with a minimum score of 2. Let n be the number of points each winning player has at the finish, p be the number of points of the loser, and d be the number of deals. The solution must satisfy

$$3 \times n + p = 26 \times d, \text{ where } p \geq 100 \text{ and } n < 100$$

Consider $d = 4$, so $3 \times n + p = 104$. Possible solutions for (n, p) are (0, 104) and (1, 101). But one of the 3 winning players must get at least one point each deal. With 4 deals, at least one of the winning players must have more than 1 point, so no solution.

Consider $d = 5$, so $3 \times n + p = 130$. Possible solutions for (n, p) are

(0, 130), (1, 127), (2, 124), (3, 121), . . . , (10, 100). Again one of the 3 winning players must get at least one point each deal. With 5 deals, at least one of the winning players must have more than 1 point, so this eliminates n = 0 and n = 1. Thus the minimum score of the three winners is 2.

For example, this could be accomplished in 5 deals with the four players having the resulting scores of:

$$(25, 1, 0, 0) \text{ from 1st deal}$$
$$(25, 0, 1, 0) \text{ from 2nd deal}$$
$$(25, 0, 0, 1) \text{ from 3rd deal}$$
$$(24, 1, 1, 0) \text{ from 4th deal}$$
$$(25, 0, 0, 1) \text{ from 5th deal}$$

$$(124, 2, 2, 2) \text{final score}$$

M/J 2. The following thorough solution is from Charles Wampler.

Let A and B be the ages of the parent and child, and suppose the parents age is a two-digit number written XY. Then: $A = 10X + Y$ and $B = 10Y + X$. So $A - B = 9(X - Y) = N > 0$, where N is their age difference, positive because the parent is older than the child. X, Y, and N are all integers, so N must be a multiple of 9, and $X > Y$. Also, X and Y are single digits, i.e., they range from 0 to 9.

There is a slight ambiguity in the problem, namely, do we allow $Y = 0$? If we allow this, then, for example, a parent 18 years older than a child will have reversals beginning at $(A, B) = (20, 02)$ and recurring every 11 years for $(A, B) = (31, 13), (42, 24), \ldots, (97, 79)$.

Let's rule out $Y = 0$, as no one normally writes the age of a youngster as "02". Then, the age reversals can happen for any value of $D = (X - Y)$ from 1 to 8, with Y running 1 to $9 - D$ and $X = Y + D$. If the parent is $N = 9D$ years older than the child, they will experience $9 - D$ reversals at ages given by

$$(X, Y) = (D + 1, 1), (D + 2, 2), \ldots, (9, 9 - D).$$

Note that Edward and his daughter have an age difference of $N = 27 = 9*3$. So their full list of six reversals are $(41,14)$, $(52,25)$, $(63,36)$, $(74,47)$, $(85,58)$, and $(96,69)$. Since Edward's son is three years older than the daughter, that parent-child age difference is not a multiple of 9, so they will have no reversals.

Finally, in these calculations, ages increment by 1 on a person's birthday. If the parent's age is a multiple of 9 on the day the child is born, then the reversals will happen on the child's birthday; otherwise, the parent's age on the day the child is born must be one less than a multiple of nine, and the reversals will happen on the parent's birthday.

M/J 3. Alan Stern sent us the following response, giving what I believe was Gardner's expected solution.

Part 1. If the older child's gender is capitalized and the younger child's gender is lower case, there are four combinations possible for the two children: Bb, Bg, Gb and Gg. For three of these cases, the parent can say that he has a boy, but only one case consists of two boys, so the probability of two boys is 1/3.

Part 2: For ease of notation, I'll denote the day of birth by a number (Sunday is 1, Monday is 2, etc.). Then the older child can be B1 to B7 or G1 to G7 (14 cases) and the younger child can be

b1 to b7 or g1 to g7 (14 cases). There are 14 x 14 = 196 combinations of older child/day and younger child/day.

Of these combinations, 27 contain a B3 and/or a b3. They are the following 28 pairs minus 1 for double counting (B3,b3): (B3,b1), ..., (B3,b7), (B3,g1), ..., (B3,g7), (B1,b3), ..., (B7,b3), (G1,b3), ..., (G7,b3).

Of these 27 combinations, 14 are a boy and a girl: (B3,g1), ..., (B3,g7), (G1,b3), ..., (G1,b3). This leaves 13 combinations of two boys. The required probability is therefore 13/27

Sam Ribnick offers a "visual explanation" that I have placed on the Puzzle Corner website.

Let me conclude with Timothy Chow's comment that as stated there is not enough information given. Chow agrees that the "intended" answers are as given above, but notes that Gardner himself realized there is ambiguity. Chow points us at http://en.wikipedia.org/wiki/Boy_or_Girl_paradox for details (including a Bayesian analysis). Other readers also noted ambiguities.

Better Late Than Never

2014 N/D 1. James Shearer sent the following improved solution.

Consider the position where White has pawns on a2, b6, c2, c6, c7, d2, d6, h3 and his king on f1, Black has pawns on f2, f3, g3, h7, a bishop on g1, a knight on h1 and his king on c8. Suppose it is Black to move. Then Black delays mate for as long as possible by moving his bishop back and forth between g1 and h2. White can move the h3 pawn to h6 (3 moves), the c2 and d2 pawns to c5 and d5 (6 moves) and the a2 pawn to a8 (6 moves). But then White has to mate Black on his next move. So mate can be delayed until white's 16th move but no longer. Alternative continuations will lead to earlier mates.

Other Responders

Responses have also been received from F. Albisu, C. Barbehenn, R. Batzinger, S. Berkenblit, J. Boynton, J. Chandler, T. Chase, G. Coram, D. Diamond, J. Feil, M. Fineman, R. Giovanniello, P. Groot, J. Hardis, O. Helbok, P. Higbie, Y. Hinuma, A. Hirshberg, W. Lemnios, D. Loeb, J. Mackro, J. Martinez, D. McMahon, T. Mita, F. Model, R. Morgen, S. Nason, B. Norris, J. Norvik, T. Oklay, A. Ornstein, F. Pollitz, J. Prussing, P. Rauch, A. Raynus, K. Rosato, G. Ruden, D. Truesdale, L. Wagner, C. Wampler, J. Wolf, and S. Zalkin.

Proposer's Solution to Speed Problem

Let $P = x\hat{\ }x\hat{\ }x\hat{\ }x\hat{\ }\ldots$. Then $P = 2$ and $P = x\hat{\ }P$. Hence $x = \sqrt{2}$.