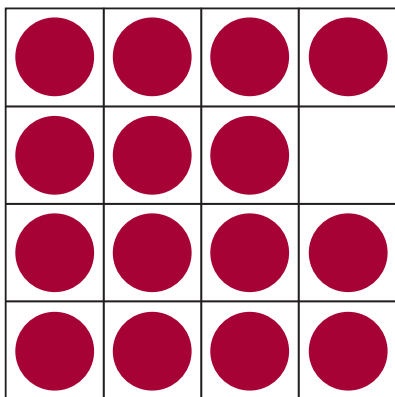


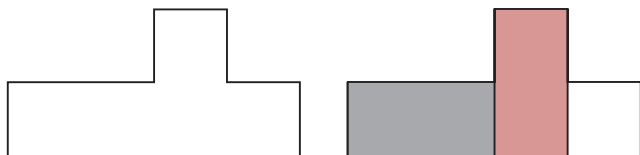
Oops, $RH \neq RH$. In the November/December issue I referred to “the late Richard Hess,” which Hess was surprised to read. Quoting Mark Twain, Hess notes, “The reports of my death have been greatly exaggerated.” I confused Richard Hess with Robert High and sincerely apologize.

Problems

M/A 1. Rocco Giovanniello loves his winks! This time we have a four-by-four grid with positions initially filled as shown below. Each move consists of having one wink jump over another (vertically or horizontally) adjacent wink, removing the latter. The goal is to find a series of moves ending with just one wink on the board.



M/A 2. Richard Hess, Robert Wainwright, and Yoshiyuki Kotani note that the pentomino on the left can easily be 80 percent covered with two congruent tiles as shown on the right. They want you to find another tile such that two of them can cover more than 86 percent of the pentomino. The tiles may not overlap each other or the pentomino border. One of the tiles may be turned over to achieve the covering.



M/A 3. Howard Stern is concerned that someone is in his seat. An airplane with 100 seats is fully booked. Every passenger is assigned a seat. However, the first passenger to board ignores his assignment and chooses a seat at random. The second and all subsequent passengers sit in their assigned seat if it is unoccupied when they board. If their seat is occupied, they choose an empty seat at random. I board the plane last and sit in the one remaining seat. What are the odds I am sitting in my assigned seat?

Speed Department

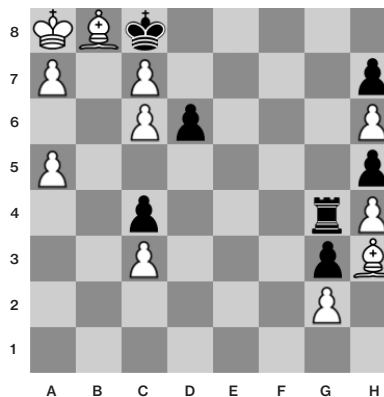
Sorab R. Vatcha wonders what the following seven words have in common: assess, banana, dresser, grammar, potato, revive, uneven.

Solutions

N/D 1. Duffy O’Craven offers what he describes as a “delicious variation upon retrograde chess problems.”

Helplessmate is a position where all legal continuations lead to checkmate (assuming neither player resigns and they do not agree to a draw). Show a helplessmate position in which the longest continuation is maximal.

Richard Stanley writes: “Such problems have been considered before. In 1994 Noam Elkies composed the position below, which requires seven moves to mate. Elkies also constructed a position (which I cannot recall) with many promoted men on the board that requires ten moves.” (See the Puzzle Corner website for Stanley’s examples of an interesting variant to this problem.)



N/D 2. Another “logical hat” puzzle previously sent to us by (the very much alive) Richard Hess. In these puzzles, logicians are wearing hats with numbers. The logicians see the number on every other hat, but not on their own. Each logician’s reasoning is error-free (and each knows that the other logicians’ reasoning is error-free).

In this particular problem there are five logicians, A, B, C, D, and E, and the unusual property that E is blind. There are nine slips of paper: five slips have a 7 written on them and the remaining four have an 11. Five of the slips, chosen at random, are placed on the logicians’ hats; the remaining four slips are hidden.

The logicians make the following statements. A, B, C, D state (in turn), “I don’t know my number.” Then E states, “I know my number.” What is E’s number?

Jay Sinnett tells us that this puzzle is a fairly easy one, once you realize that it is *not* in the category of “Find the bogus coin in seven weighings or less.” In other words, in this case the solution is not “general”; it’s specific to a particular set of cases.

If A saw only 11s on the hats of the other four logicians, then he would know that his hat must have a 7, because there are only four 11s available. Therefore, we know that A saw at least one 7 in the group B-C-D-E.

Now when it's B's turn to speak, he knows that A saw at least one 7 in the group B-C-D-E. So if B saw only 11s on C, D, and E, then he would know that A saw only one 7, and it's on his own (B's) hat. But B says he's not sure—so B saw at least one 7 in the group C-D-E.

Similarly, when it's C's turn to speak, he knows that B saw at least one 7 in C-D-E, so if C saw only 11s on D and E, then C would know that B saw only one 7, and it's on his own (C's) hat. But C says he's not sure—so C saw at least one 7 in the group D-E.

When it's D's turn to speak, he knows that C saw at least one 7 in the group D-E, so if D saw an 11 on E's hat, then D would know that he (D) must be wearing a 7. But D says he's not sure—so D saw a 7 on E's hat.

Therefore, E can say with certainty, "I have a 7 on my hat."

N/D 3. J. D. R. Kramer sent us the following famous old problem, originally attributed to Lewis Carroll.

Some men sat in a circle, so that each had two neighbors. Each had a certain number of coins. The first had one coin more than the second, who had one coin more than the third, and so on. The first gave one coin to the second, who gave two coins to the third, and so on, each giving one coin more than he received, for as long as possible. There were then two neighbors, one of whom had four times as much as the other.

How many men were there? And how many coins did the poorest man have at first?

Terence Sim solved this problem as well as several cases of a generalization where at the end the multiple was m , not two. Sim writes:

"The answer is: seven men, and the poorest man starts off with two coins.

"Let there be n people seated clockwise in a circle. Man # n , the poorest, starts with k coins. His left neighbor, Man #1, starts with $n - 1 + k$ coins.

"To make it easier to analyze the problem, let's introduce a banker, who sits between Man # n and Man #1. The banker's role is simply to pass all the coins from Man # n to Man #1, without adding or removing coins.

"Define one round as completed when the banker receives coins from Man # n ; the next round starts when the banker passes the coins to Man #1.

"After the first round, the banker has n coins, since every man gave up one of his own coins. Man # n is left with $k - 1$ coins. Clearly, this process can go on k times, whereupon Man # n has zero coins and the banker has kn coins.

"On the $(k + 1)$ st round, everyone except Man # n gives up one coin, as before. But Man # n is unable to, since he is short of coins. Thus the coin-passing stops, and Man # n has $(k + 1)n - 1$ coins in his possession.

"At this moment, the banker has no coins, but Man #1 has $n - 2$ coins. This has to be one-fourth as many as Man # n . Thus, we get the equation: $(k + 1)n - 1 = 4(n - 2)$

"Rewrite this to get: $(k - 3)n = -7$

"Since n and k are non-negative integers, the only solution to the equation is $k = 2, n = 7$.

"To explore further, suppose the multiple when the coin-passing stops is m instead of 4. Then we get the equation $(k + 1 - m)n = 1 - 2m$.

"By inspection, we get these solutions for different values of m :

$m = 1$: no solution

$m = 2$: $k = 0, n = 3$

$m = 3$: $k = 1, n = 5$

$m = 4$: $k = 2, n = 7$

$m = 5$: $k = 3, n = 9$ OR $k = 1, n = 3$

$m = 6$: $k = 4, n = 11$

$m = 7$: $k = 5, n = 13$

$m = 8$: $k = 6, n = 15$ OR $k = 4, n = 5$ OR $k = 2, n = 3$

"Thus, for some values of m , there are multiple solutions."

Better Late Than Never

S/O SD. Joseph Horton notes that the tetrahedron is indeed correct if the cut surfaces are not parallel or if we were aiming to cut off the largest possible number of corners. But as the problem is stated, there is a way to make the cut surfaces parallel, which yields an antiprismatic octahedron: "Not a very pretty shape, but there it is."

Other Responders

Responses have also been received from F. Albisu, D. Aucamp, J. Bohannon, R. Bronowitz, J. Chandler, S. Eddy, R. Giovanniello, H. Gross, J. Grossman, J. Hardis, J. Harmon, A. Hirshberg, P. Kramer, E. Kutin, W. Lemnios, D. Limbert, Z. Lu, P. McCreary, T. Mita, R. Morgen, S. Nason, E. Nelson-Melby, M. Perkins, W. Reenstra, K. Rosato, H. Safer, L. Schaidler, P. Schottler, E. Sheldon, M. Strauss, and S. Wilson.

Proposer's Solution to Speed Problem

If you move the first letter of each word to the end and read the result backwards, you get the original. Vatcha calls these quasi-palindromes.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.