It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so again.

As responses arrive, they are simply put together in neat piles, with no regard to their date of arrival. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I try to select a solution that supplies an appropriate amount of detail and includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or (especially) sent by e-mail, since these simplify typesetting.

Problems

N/D 1. Duffy O'Craven offers what he describes as a "delicious variation upon retrograde chess problems."

Helplessmate is a position where all legal continuations lead to checkmate (assuming neither player resigns and they do not agree to a draw). Show a helplessmate position in which the longest continuation is maximal.

N/D 2. Another "logical hat" puzzle previously sent to us by the late Richard Hess. In these puzzles, logicians are wearing hats with numbers. The logicians see the number on every other hat, but not on their own. Each logician's reasoning is error-free (and each knows that the other logicians' reasoning is error-free).

In this particular problem there are five logicians, A, B, C, D, and E, and the unusual property that E is blind. There are nine slips of paper: five slips have a 7 written on them and the remaining four have an 11. Five of the slips, chosen at random, are placed on the logicians' hats; the remaining four slips are hidden.

The logicians speak. A, B, C, and D state (in turn), "I don't know my number." Then E states, "I know my number." What is E's number?

N/D 3. J. D. R. Kramer sent us the following famous old problem (the source will be revealed with the solution two issues hence).

Some men sat in a circle, so that each had two neighbors. Each had a certain number of coins. The first had one coin more than the second, who had one coin more than the third, and so on. The first gave one coin to the second, who gave two coins to the third, and so on, each giving one coin more than he received, for as long as possible. There were then two neighbors, one of whom had four times as much as the other.

How many men were there? And how many coins did the poorest man have at first?

Speed Department

Fred Cann says you can do this in your head; I would permit a pencil and paper. Start with a unit-length cube and remove a tetrahedron corner by cutting the diagonals of the three adjacent faces. How many corners can you cut off this way, and what, if anything, is left of the cube?

Solutions

J/A 1. Larry Kells is an economical guy, at least when it comes to high-card points. He wonders, what is the lowest number of points a bridge partnership can have and still be able to make 7 no-trump? You choose the cards for declarer and dummy but must succeed against any distribution of the remaining cards between the opponents and best defense. What about 6 or 3 no-trump?

Doug Foxvog, Jerrold Grossman, Ed Sheldon, and Richard Thornton agree with the proposer that the answers are 19, 18, and 15 points, respectively. Foxvog writes:

"For a bridge partnership to win 7 no-trump, it must win the first trick no matter what suit is led. Since it cannot trump, it must have an ace in the led suit. Once it wins the first trick it could run a long suit for the rest of the tricks, but this can only be done if the opponents don't have Kx or Qxx in the wrong hand. The first can be prevented if the partnership has a 12-card suit (and thus three more points—the Q and J). The second can be prevented if the partnership has an 11-card suit including the K (which is also three points). Thus the minimum number of high-card points that can guarantee 7 no-trump against any defense is 19 (four aces plus either a king or a queen and jack).

"The first alternative is illustrated by

Dummy	**Declarer**
♠	♠ A Q J × × × × × × × × ×
♥ × × × ×	♥ A
♦ A × × ×	♦
♣ A × × ×	♣

"For the partnership to win 6 no-trump, it can lose the first trick if it is guaranteed the second. This can be managed if the dummy has the same long suit as above, but the declarer has Kx in the single suit for which the partnership is missing the ace (it must be the declarer; otherwise the K can be finessed). After the opponents take their ace, the remaining tricks are easily won as above. If defense does not take the ace on the first trick, an overtrick is made. For example:

Dummy	**Declarer**
♠ A K × × × × × × × ×	♠
♥ A	♥ × × × ×
♦ A	♦ × × × ×
♣	♣ K × × × ×

"For a partnership to win 3 no-trump, it can allow the opponents to take four tricks—all the points in one suit, if it has a protected 10. Giving the partnership three aces plus a king is 15 points, as illustrated below. To allow opponents to take two aces would require that blocking kings be nonfinessible—by having a queen under them, requiring five points to block a suit instead of just four with an ace."

Dummy	Declarer
♠	♠ A K × × × × × × × × ×
♥ × × × ×	♥ A
♦ × × × ×	♦ A
♣ 10 9 8 7 6	♣

J/A 2. Sorab Vatcha offered a "symbolic problem," which I slightly modified to the following. What is the longest English word consisting entirely of the symbolic names of (possibly repeated) elements in the periodic table? For example, a six-letter solution is "sinner": Si N N Er, or silicon, nitrogen, nitrogen, erbium. For consistency, an English word is one accepted by www.merriam-webster.com.

I was pleased with my requirement of a fixed dictionary, but I now see that the question is still ambiguous (though not very): while www.merriam-webster.com "accepts" the word "nonrepresentationalisms," it "reduces" it to "nonrepresentational." That is, when you type the former, it gives you the definition of the latter. Since I just asked for acceptance, not a definition, I am allowing it. Also, I see that in addition to the free version of merriam-webster.com that I use (and intended), there is an "unabridged" version, which requires a subscription. The words "table," "dog," and "cat" are in the free version, but "tables," "dogs," and "cats" explicitly refer you to the unabridged version.

In summary, the "official" solution is the 23-letter word "NONRePReSeNTaTiONAlISmS," found by Michael Branicky and Zoltan Mester, but I can understand why one might consider it only the 19-letter word "NONRePReSeNTaTiONAl."

J/A 3. John Oehrle has a problem that is simple to state mathematically but (to me) looks hard to solve. However, I have often been wrong with such predictions.

Define a shuffle of 52 cards that requires the most consecutive applications to return the deck to its original order. For example, if you numbered the cards in a deck from 1 to 52, a simple shuffle might be: $1 \to 2$, $2 \to 1$, $3 \to 4$, $4 \to 5$, $5 \to 3$, and $n \to n$ for $n > 5$. This shuffle must be repeated six (the least common multiple—LCM—of 2, 3, and 1) times to return the deck to the original order. Mathematically stated, the problem asks for a partition of 52 having maximal LCM.

As was pointed out by several readers, the maximal LCM of a partition of n is known as Landau's function. It is equivalently defined as the largest order of an element of the symmetric group S_n. Information on Landau's function can be found in Wikipedia, together with the values for $n \leq 47$.

I received a number of fine solutions. Space limits me to including just a few.

At one extreme, Leo Stein rolled out the heavy artillery (Mathematica) and produced this one-line program:

L[*n*_]: = *Max*@*Apply*[*LCM*, *IntegerPartitions*@*n*, 1]; *L*[52] yields 180,180. Just adding one joker doubles the answer; i.e., *L*[53] yields 360,360.

Ken Zeger points us at /oeis.org/A000793/b000793.txt, which lists $L[n]$ for all $n \leq 10{,}000$. In particular, $L[10{,}000]$ has 137 digits.

Dave Blackston writes that for a given N (52 in this case), we are looking for a set of numbers whose sum is at most N and whose LCM is as large as possible. (If the sum is less than N, we may add a number of 1s to get the sum to N.) Now, note that if A and B are different and both greater than 1, then $AB > A + B$. This means that if there is an element of our partition that is not a prime power, then we may replace it with the prime powers occurring in its factorization, reducing the sum but maintaining the LCM. This means that there is an optimal partition whose elements are all prime powers or 1. Hence we may restrict our search to partitions consisting only of prime powers.

This is actually a straightforward dynamic programming problem where we use the following recurrence. Let $F(N, P)$ be the maximum LCM for a partition of N where all primes represented are at most P. Then if P is prime,

$$F(N, P) = \max \{ F(N, P-1)\} \cup \{P^i \times F(N - P^i, P-1) \mid P^i \leq N\}$$

This gives [1, 1, 1] + [4, 5, 7, 9, 11, 13] for an LCM of 180,180.

Terence Sim writes (essentially) this partition explicitly as (1), (2), (3), (4,5,6,7), (8 – 12), (13 – 19), (20 – 28), (29 – 30), (40 – 52).

Other Responders

Responses have also been received from J. Chandler, J-P. Garric, J. Harmse, H. Hodara, P. Kebabian, N. Markovitz, A. Ornstein, A. Prakash, B. Rhodes, K. Rosato, M. Strauss, M. Thattai, S. Vatcha, and D. Worley.

Proposer's Solution to Speed Problem

4. Viewing a face of the original cube, they are the top left front, top right rear, bottom right front, and bottom left rear. What remains is a regular tetrahedron with edge length $\sqrt{2}$.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.