It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a large queue of regular problems and a comfortable supply of speed problems. However, I could use some more game-inspired problems. For example, the first problem in this issue is sudoku-inspired.

Problems

M/J 1. Apo Sezginer is fond of sudoku and is interested in certain transformations (defined below) of the 9x9 grid. Any sudoku solution is mapped to other solutions by these transformations. The question is whether there are two solutions such that none of the transformations maps the first to the second.

To define the transformations, recall that the 9x9 grid A is normally indexed via

$$A[p,q]; \quad p = 1, 2, \cdots, 9; \quad q = 1, 2, \cdots, 9$$

Instead we wish to use four indices, each ranging from 1 to 3. Specifically, we define the four-index representation in terms of the standard two-index form via

$$A[M,N,m,n] = A[(M-1)\times 3 + m, (N-1)\times 3 + n]$$

The transformations are:

Permute M = 1,2,3 keeping other indices unchanged.
Similarly permute N = 1,2,3; m = 1,2,3; and n = 1,2,3.
Permute the arabic numerals.

M/J 2. The late Bob High had sent us a number of problems, including many cryptarithmetic examples in which one substitutes a digit for each letter so that the result is a true arithmetic statement. This issue's offering features "a bit of Spanish."

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 POCO
 POCO
 POCO
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 POCO
 POCO
 POCO
 POCO
 POCO
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MUCHO
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M/J 3. Fred Davidson recalls the familiar "birthday paradox," where we calculate that if 23 people are chosen at random, then there is about a 50 percent chance that at least two have the same birthday (ignoring leap years, and assuming all birthdays are equally likely).

Davidson, however, has chosen 365 people at random and wants to know how many distinct birthdays would occur on average (same assumptions as above).

Speed Department

As a reminder of days gone by—for me, the 1950s and '60s—here is Michael Brill's takeoff on a standardized-test analogy question. What should replace XX in the following?

Billy : Captain :: Stan : XX

Solutions

J/F 1. Larry Kells asks us about the "Law of Total Tricks" and, in the best spirit of Thoreau's "Civil Disobedience," wants to know how badly we can violate this law (really a heuristic).

The law states that the expected number of tricks N-S can take declaring in their longest trump suit, added to the number of tricks E-W can take declaring in their longest suit, is approximately equal to the number of trumps N-S have in their suit, added to the the number of trumps E-W have in theirs.

What is the greatest possible excess of total tricks over total trumps (with best play on both sides)? What is the greatest possible deficit? Assume the declarer is whichever one can make the most tricks with their suit as trumps.

Mark Bolotin first gives us a hand with a total excess of 16 tricks over trumps.

North
♠ 5 4 3 2
♥ A K Q J 10
♦
♣ 9 8 7 6

West
♠
♥ 9 8 7 6
♦ A K Q J 10 9 8 7 6
♣

East
♠
♥ 5 4 3 2
♦ 5 4 3 2
♣ A K Q J 10

South
♠ A K Q J 10 9 8 7 6
♥
♦
♣ 5 4 3 2

South as declarer makes seven hearts. East as declarer makes seven clubs. The difference is 16. It is impossible to make seven in a suit (without revoke) with only four combined trumps, as one defender will have a long trump. Thus you cannot exceed an excess of 16 with grand slams.

Bolotin then produces a deficit of 16 tricks over trumps.

North
♠ 5 4 3 2
♥ A K Q J 10
♦
♣ 9 8 7 6

West
♠ 6
♥ 9 8 7 6
♦ A K Q J 10 9 8 7
♣

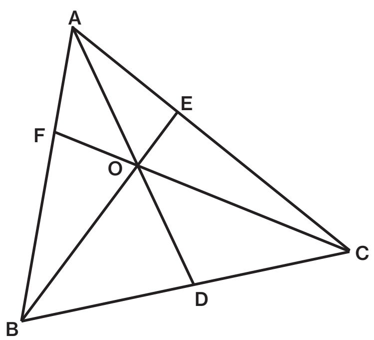
East
♠
♥ 5 4 3 2
♦ 5 4 3 2
♣ A K Q J 10

South
♠ A K Q J 10 9 8 7
♥
♦ 6
♣ 5 4 3 2

North-South can be held trickless with clubs as trump, no matter which one is declarer. Likewise, North-South can run the first 13 tricks on defense against a heart contract declared by either East or West.

J/F 2. David Kramer enjoyed reading a book entitled *Lewis Carroll in Numberland*. In particular, he found it contained several interesting problems, including the following.

Given a triangle ABC, draw lines from each vertex to the opposite side so that all three lines meet. Label the points D, E, F, and O as shown in the diagram below. You are to find the ratio DO/DA in terms of the two ratios EO/EB and FO/FC.



Jack Bross notes that what we are basically doing is applying barycentric coördinates. He believes that the easiest way to derive the result is to think of the problem in terms of area.

First note that the ratio DO/DA is the same as the ratio of the areas of triangle $\triangle BOC$ and the large triangle $\triangle ABC$. This is so because the two triangles have the common base BC, and have heights respectively $DO \sin(\measuredangle ODC)$ and $DA \sin(\measuredangle ODC)$.

Similarly, the ratio EO/EB equals the ratio of the areas of $\triangle AOC$ and $\triangle ABC$. Finally, the ratio FO/FC equals the ratio of the areas of $\triangle AOB$ and $\triangle ABC$.

From the diagram we see that the area of $\triangle ABC$ is the sum of the areas of $\triangle BOC$, $\triangle AOC$, and $\triangle AOB$. Dividing both sides of this equality by the area of $\triangle ABC$ gives

$$\frac{\text{area } \triangle ABC}{\text{area } \triangle ABC} = \frac{\text{area } \triangle BOC}{\text{area } \triangle ABC} + \frac{\text{area } \triangle AOC}{\text{area } \triangle ABC} + \frac{\text{area } \triangle AOB}{\text{area } \triangle ABC}$$

or

$$1 = \frac{DO}{DA} + \frac{EO}{EB} + \frac{FO}{FC}$$

as desired.

Henry Hodara notes that this result follows from the classical theorems of Menelaus and Ceva (who just missed each other by 1,600 years).

Better Late Than Never

Y 2013. Ermanno Signorelli notes that $5 = 20/(1 + 3)$ improves on the given solution and corrects $59 = 20 \times 3 - 1$. Tom Keske, George St. Pierre, and Jordan Wouk correct my other error, $10 = 10 \times (3 - 2)$.

J/F SD. John Astolfi and George Fischer gave detailed explanations (perhaps I erred in making this a speed problem).

Other Responders

Responses have also been received from C. Dailey, R. Lipes, P. Manglis, M. Mann, and S. Vatcha.

Proposer's Solution to Speed Problem

Thomas. The rationale being

Billy (the Kid) : Captain (Kidd) ::
Stan (the Man) : Thomas (Mann)

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.