This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 4) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2013 yearly problem is in the "Solutions" section.

Problems

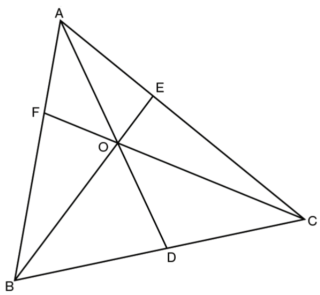
Y2014. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 4 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 4 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Larry Kells asks us about the "Law of Total Tricks" and, in the best spirit of Thoreau's "Civil Disobedience," wants to know how badly we can violate this law (really a heuristic).

The law states that the expected number of tricks N-S can take declaring in their longest trump suit, added to the number of tricks E-W can take declaring in their longest suit, is approximately equal to the number of trumps N-S have in their suit, added to the number of trumps E-W have in theirs.

What is the greatest possible excess of total tricks over total trumps (with best play on both sides)? What is the greatest possible deficit? Assume the declarer is whichever one can make the most tricks with their suit as trumps.

J/F 2. David Kramer enjoyed reading a book entitled *Lewis Carroll in Numberland*. In particular, he found it contained several interesting problems, including the following.



Given a triangle ABC, draw lines from each vertex to the opposite side so that all three lines meet. Label the points D, E, F, and O as shown in the diagram. You are to find the ratio DO/DA in terms of the two ratios EO/EB and FO/FC.

Speed Department

Two teams, the Slow Starters and the Late Chokers, play each other annually in a best-of-seven series. The Slow Starters always lose game 1, and if the series reaches 3–3, the Late Chokers always lose game 7. Otherwise, the teams are equally likely to win a given game. Which team is more likely to win the series?

Solutions

Y2013. Finally, a year with four distinct digits! I received a number of fine solutions; the one below is from Avi Ornstein.

$1 = 1^{203}$	$19 = 20 - 1^3$	$49 = (10 - 3)^2$
$2 = 2 + 0 \times 13$	$20 = 20 \times 1^3$	$50 = (2 + 3) \times 10$
$3 = 3 \times 1^{20}$	$21 = 20 + 1^3$	$51 = 20 + 31$
$4 = 10 - 2 \times 3$	$22 = 32 - 10$	$57 = (20 - 1) \times 3$
$5 = 10 - 3 - 2$	$23 = 20 + 1 \times 3$	$58 = (30 - 1) \times 2$
$6 = (2 + 0 \times 1) \times 3$	$24 = 20 + 1 + 3$	$59 = 20 + 3 - 1$
$7 = 20 - 13$	$26 = (2 + 0) \times 13$	$60 = 20 \times 1 \times 3$
$8 = 10/2 + 3$	$27 = 30 - 2 - 1$	$61 = 20 \times 3 + 1$
$9 = 10 - 3 + 2$	$28 = 30 - 2 \times 1$	$62 = (30 + 1) \times 2$
$10 = 10 + (3 - 2)$	$29 = 31 - 2 + 0$	$63 = (20 + 1) \times 3$
$11 = 10 + 3 - 2$	$30 = 30 \times 1^2$	$65 = 130/2$
$12 = 12 + 3 \times 0$	$31 = 31 + 2 \times 0$	$67 = 201/3$
$13 = 23 - 10$	$32 = 32 \times 1 + 0$	$70 = 210/3$
$14 = (10 - 3) \times 2$	$33 = 20 + 13$	$80 = 20 \times (1 + 3)$
$15 = 10 + 3 + 2$	$34 = 102/3$	$90 = 30 \times (2 + 1)$
$16 = 20 - 1 - 3$	$36 = (10 + 2) \times 3$	$97 = 10^2 - 3$
$17 = 20 - 1 \times 3$	$40 = 120/3$	$99 = 102 - 3$
$18 = 20 + 1 - 3$	$42 = 12 + 30$	

S/O 1. I received multiple solutions to Rocco Giovanniello's wink problem starting with a 4 × 6 board, with the square (4,3) empty and the others containing a wink. As usual, the goal was to find a sequence of horizontal and vertical jumps so that only one wink remained. The solution below, which results in the original empty square being the only one occupied, is from Ted Mita.

(4,1) → (4,3)	(3,1) → (3,3)	(6,3) → (4,3)
(2,1) → (4,1)	(2,4) → (2,2)	(4,4) → (4,2)
(2,3) → (2,1)	(4,4) → (2,4)	(3,2) → (5,2)
(1,1) → (3,1)	(1,4) → (3,4)	(6,2) → (4,2)
(1,3) → (1,1)	(3,4) → (3,2)	(6,1) → (4,1)
(4,1) → (2,1)	(2,2) → (4,2)	(4,1) → (4,3)
(1,1) → (3,1)	(5,2) → (3,2)	
(4,3) → (2,3)	(6,4) → (4,4)	

S/O 2. I received a number of beautiful solutions to Alan Faller's Monhegan Island sailing problem, but I didn't see how I could not print this one from the captain of the '59 MIT sailing team, William Widnall:

"Alan asks for the correct initial sailboat heading relative to the wind direction to reach a point (let's call it the mark) to windward that initially is in a direction 20° away from dead upwind, assuming that he will change direction once and that the boat's speed potential as a function of heading is $v = V \sin(2\pi \beta/360)$, where V is a constant and β is the angle in degrees between the boat's heading and the upwind direction, valid for $0 \le \beta \le 90°$. Assume that the boat's speed versus heading characteristic is symmetric about the wind direction. That is, $v(\beta) = v(-\beta)$. Fig. 1 shows the boat's performance polar diagram.

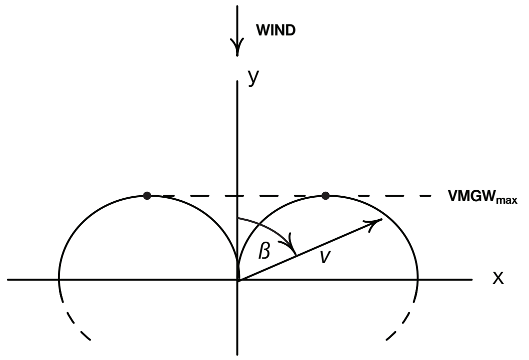


Fig. 1 Boat performance polar diagram

"Interpret 'correct heading' to mean the best initial heading to reach the mark to windward in the minimum amount of time. Assume the wind direction is constant. Then it can be shown that the minimum-time solution utilizes the two headings that equally maximize the velocity made good to windward (VMGW). Restate the performance function as a normalized speed $r = \sin\theta$ where $r = v/V$ and $\theta = 2\pi\beta/360$. The component of the boat's normalized velocity vector in the upwind direction (y direction in the xy plane of Fig. 1) is $y = r(\theta)\cos\theta = \sin\theta\cos\theta$. A maximum of y occurs where $\frac{dy}{d\theta} = (\cos\theta)^2 - (\sin\theta)^2$ equals 0. In the range from 0 to $\pi/2$, this occurs only at $0 = \pi/4$ (heading 45°). Because of the assumed symmetry in boat performance, an identical maximum value of VMGW is achieved by the heading of negative 45°.

"There are two solutions to the stated problem. See Fig. 2. To get to the desired upwind point with only one turn, initially head at either plus or minus 45° with respect to the wind. When you reach the 'lay line,' defined as the line that will take you directly to the mark while sailing the heading for best VMGW (for this boat it's the other 45° heading), turn to the other 'tack' and head for the mark.

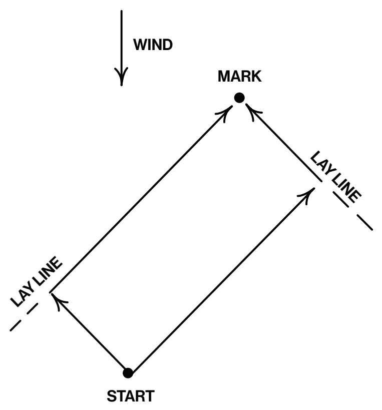


Fig 2. The two single-turn optimal tracks

"Either course gets you to the windward mark in the same minimum time. Any alternative steering strategy will take longer to climb the upwind distance to the windward mark.

"Note that the optimal headings are not a function of the given 20° initial offset of the windward mark relative to the wind. Any offset mark direction within plus or minus 45° from the wind would be reached optimally by the same strategy."

Tim Barrows augments the general case with Maine-specific wind information. John Chandler believes that the boat speed should be $\sin^2(\beta/2)$ rather than $\sin(\beta)$. Alan Faller's blog (weatherstream.wordpress.com) includes a discussion of this problem.

S/O 3. Tim Maloney wants you to show that all solutions of the complex equation

$$e^z = \frac{z-1}{z+1}$$

lie on the imaginary axis.

I apologize for originally misspelling the proposer's name.

Jim Simmonds first notes that $z = -1$ is *not* a solution. He then sets $z = x + yi$ and takes the absolute value of each side to obtain

$$e^x = \sqrt{\frac{(x-1)^2 + y^2}{(x+1)^2 + y^2}}$$

The right side is less than 1 if $x > 0$ and greater than 1 if $x < 0$; e^x is just the opposite. Thus equality holds only at $x = 0$, where the original equation is transcendental and has an infinite number of solutions satisfying $y = \cot(y/2)$.

Eric Nelson-Melby notes that the five smallest solutions for positive y are approximately 1.3065, 6.5846, 12.7232, 18.9550, and 25.2120. Apo Sezginer gave a detailed analysis of the last equation, which I have placed on the "Puzzle Corner" Web page.

Better Late Than Never

S/O SD. James Simmonds and Paul Howard note that we should have required "integer greater than 1," not simply "number." The wildest examples are complex numbers (with modulus) less than 1 whose square just has a minus sign prepended.

Other Responders

Responses have also been received from P. Cassady, J. Cohen, C. Dale, A. Forer, J. Harmse, R. Hess, D. Karlin, M. Kay, P. Kebabian, D. Kramer, I. Lai, P. Manglis, A. Mattick, C. McCallum, R. Morgen, C. Patton, F. Politz, E. Sard, M. Seidel, A. Sezginer, E. Signorelli, M. Strauss, I. Swetlitz, T. Turek, and K. Zeger.

Proposer's Solution to Speed Problem

They are equally likely to win.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.