

**H**ello, Hunter! Big doings around the Gottlieb household. This issue of “Puzzle Corner” is dedicated to Hunter David Gottlieb, our first grandchild, born July 18 to our elder son, David, and his lovely wife, Marissa. Hunter and his parents are doing well; the four grandparents and five great-grandparents are ecstatic.

I can more or less accept that I am a grandfather but find it *impossible* to look at my fetchingly beautiful bride, Alice, and think “grandma.”

Bill Reenstra reports that after reading the column for about 40 years he finally submitted a solution, one written “on the back of an envelope ... received from MIT thanking [him] for a contribution to the Annual Fund.” Reenstra believes that specific envelope aided him in solving the problem and suggests using such envelopes as “an aid to other followers of this column,” a suggestion I am confident the alumni fund would support.

It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent by e-mail, since these simplify typesetting.

**Problems**

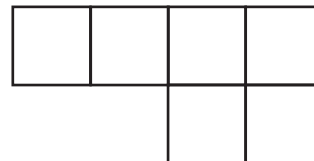
**N/D 1.** David Griffel proposes a new kind of puzzle he calls THIRDS, in which you are to use the letters from the third of the alphabet shown to replace the dashes and produce a standard English word. A letter can be used more than once. Here are three such puzzles.

- RR--- (ABCDEFGH)
- D--E-T (JKLMNOPQ)
- E--I-O-- (RSTUVWXYZ)

**N/D 2.** Unfortunately, Tom McNelly’s kitchen clock has a defect: the minute and hour hands are indistinguishable. He worries that some configurations of the hands are ambiguous—that is, they could represent two different times. Can this occur, and if so, at what times?

**N/D 3.** Another “Modest Hexominoes” puzzle from Richard Hess and Robert Wainwright, in which you are to design a connected tile so that *n* of them cover the maximum area of a given hexomino. The tiles must be identical in size and shape and may be turned over so that some are mirror images of the others.

They must not overlap each other or the border of the hexomino. For the current problem, you are to cover at least 83 percent of the hexomino below with two tiles.



**Speed Department**

Sorab Vatcha was never able to afford an entire keyboard. He asks for the longest English word that can be formed using the letters from only one row of the QWERTY keyboard.

**Solutions**

**J/A 1.** David Porter, having been dealt five spades, was pleased to hear his partner open with 1 no-trump and his opponents remain silent. Porter knows that this means his partner’s distribution was either 4-3-3-3, 4-4-3-3, 5-3-3-2, or 5-4-2-2. What are the probabilities that the partner has exactly two spades, three spades, four spades, or five spades?

Jorgen Harmse notes that one difficulty with probability problems is figuring out what is known. For example, suppose that Porter has S Axxxx H AK D AKJ C AKQ and that 1 no-trump shows 12 to 14 HCP. Then his partner must have all the outstanding high cards and therefore cannot have a doubleton in spades. To avoid such problems, Harmse assumes only that Porter has exactly five spades (and possibly more of another suit) and that his partner has one of the distributions given. Different assumptions yield different solutions.

Since the problem (as rephrased) does not hinge on Porter’s other eight cards, consider ways to divide eight spades and 39 other cards so that Porter has eight of the non-spades, his partner has 13 cards, and his opponents have 26 cards (ignore the division of cards between the opponents). The number of ways for Porter’s partner to have *s* spades, *h* hearts, *d* diamonds, and *c* clubs is

$$f(s, h, d, c) = \binom{8}{s} \binom{13}{h} \binom{13}{d} \binom{13}{c}$$

Focusing on cases that give his partner the no-trump distribution will then yield the answers. I break down the cases given according to where spades appear in the distribution. For example, 5(S)-3-3-2 means five spades, a doubleton, and three each of the other two suits.

- 5(S)-4-2-2 can occur in  $3f(5, 4, 2, 2) \approx 5.76516 \cdot 10^{15}$  ways.
- 5-4(S)-2-2 can occur in  $3f(4, 5, 2, 2) \approx 9.624097 \cdot 10^{15}$  ways.
- 5-4-2(S)-2 can occur in  $6f(2, 5, 4, 2) \approx 3.747867 \cdot 10^{16}$  ways.
- 5(S)-3-3-2 can occur in  $3f(5, 3, 3, 2) \approx 8.455568 \cdot 10^{15}$  ways.
- 5-3(S)-3-2 can occur in  $6f(3, 5, 3, 2) \approx 4.140501 \cdot 10^{16}$  ways.
- 5-3-3-2(S) can occur in  $3f(2, 5, 3, 3) \approx 2.748436 \cdot 10^{16}$  ways.
- 4(S)-4-3-2 can occur in  $6f(4, 4, 3, 2) \approx 3.920929 \cdot 10^{16}$  ways.
- 4-4-3(S)-2 can occur in  $3f(3, 4, 4, 2) \approx 2.875348 \cdot 10^{16}$  ways.

4-4-3-2(S) can occur in  $3f(2, 4, 4, 3) \approx 3.817272 \cdot 10^{16}$  ways.  
 4(S)-3-3-3 can occur in  $f(4, 3, 3, 3) \approx 9.584492 \cdot 10^{15}$  ways.  
 4-3(S)-3-3 can occur in  $3f(3, 4, 3, 3) \approx 4.217176 \cdot 10^{16}$  ways.

It follows that the probabilities are approximately these: two spades, 0.3579802; three spades, 0.389894; four spades, 0.2027662; five spades, 0.0493596.

The expected number of spades increases from 2.2 to 2.9 when Porter's partner bids no-trump, presumably because this rules out big holdings in other suits.

**J/A 2.** Ermanno Signorelli has one for all you wordsmiths and Scrabble players. Assemble a list of English words, using the smallest total number of letters, such that the alphabet is contained, in reverse alphabetical order, in the letters of the words. As an example, Signorelli offers aZYgos, ... ItcHinG, FED, CaBAL.

Once again the answer depends on the exact rules, specifically what dictionary is considered the list of words. Joel Sokel gave three solutions, using different dictionaries, but in the end I decided to agree with him when he wrote, "Finally, since Mr. Signorelli specified Scrabble players, here's my favorite, a 37-letter solution using only words in the official Scrabble tournament dictionary: za, pYX, aW, VaU, TSaR, Qi, oP, oN, MeL, KoJI, HaG, FE, DoC, BA."

I should add that I am a miserable Scrabble player and most of Sokel's words were unfamiliar to me. However, to my surprise the Scrabble dictionary is online. A quick check confirmed that all the above are indeed (Scrabble) words.

An extreme example of "dictionary dependence" is Ted Mita's observation that Merriam-Webster's Collegiate Dictionary, 11th edition, defines each single letter written alone as a noun (not an abbreviation), so there is a simple 26-character solution. However, I agree with Mita that this should be considered "a trivial, party-pooing non-solution."

**J/A 3.** Phil Lally reports that Jack and Jill, after the episode with the hill, decided to go shopping. They purchased several items, each costing a whole number of dollars (no cents). Their total expenditure was \$207. One of the items cost \$1; the others had dollar prices that are prime numbers. The sum of the digits of the price of one item was 7. Furthermore, when the prices are written down, each of the digits 1 through 9 is used exactly once. What were the prices?

Naomi Markovitz proved as follows that \$1, \$2, \$5, \$43, \$67, and \$89 is the unique solution.

"Since the digit 1 is used in \$1, there are no three-digit prices. The only two-digit prime with a digit sum of 7 and not using the digit 1 is 43. The even digits 6 and 8 must each be the tens digit of a two-digit prime, and only 7 and 9 could be the corresponding ones digit. The remaining digits 2 and 5 are each prime, and indeed the sum is 207."

A few responders noted that the problem as stated is over-specified; for example the requirement that "the digit sum of one item is 7" can be omitted.

## Better Late Than Never

**M/A 2.** Jean-Paul Garric, Everett Kittredge, Robert Park, and Eugene Sard note that at all times (not just at final equilibrium), the medallion's coordinates lie on an ellipse. Then the equilibrium solution occurs where the ellipse is horizontal.

**M/A 3.** Timothy Chow, James Russell, and James Shearer believe that the generalization to inverting  $S$  signals is faulty since the resulting (combinational) circuits have loops.

**M/J 2.** Timothy Maloney's solution has been added to the website. He also points out that node (3,1) in the solution diagram should be labeled  $3/5$  and not  $2/5$ .

**J/A SD.** Oh my. This generated considerable commentary, some of it involving open mathematical questions. First, several readers complained that the fact that the set of (the positions of) 1s (in the binary expansion of pi) and the set of 0s are each infinite doesn't imply the sets are of the same size. For example, an expansion of .00100100101 ... has twice as many 0s as 1s.

First, let me repeat that this was to be a speed problem. But second, I would point out that any two countably infinite sets are in fact of the same "size" (technically, the same "order of infinity"). For example, the set of positive integers, the set of all integers, the set of all primes, and the set of all rational numbers are of the same order. In contrast, the set of real numbers is of a higher order.

Other readers mentioned that one could ask if the ratio of 1s to 0s in the first  $n$  bits tends to 1 as  $n$  increases (if so, the number being expanded is called "normal"). Apparently it is only conjectured that pi is normal. I did not know about this concept of normality. Pretty heady stuff for a speed problem.

## Other Responders

Responses have also been received from E. Avgoustiniatos, A. Braver, J. Brennan, L. Cangahuala, H. Chiel, W. Cluett, S. Eddy, S. Eriksen, L. Fattal, J. Feil, E. Field, M. Fineman, I. Forys, E. Friedman, S. Gaither, M. Gennert, R. Giovannello, Q. Goldstein, P. Groot, J. Grossman, J. Hardis, O. Helbok, A. Hirshberg, J. Horton, P. Hutchins, J. Iler, S. Korb, L. Kohnfelder, L. Kruglyak, F. Kuttner, D. Lampart, W. Lemnios, M. Lipman, V. Luchangco, M. Mann, M. Marinar, N. Markovitz, G. Marks, W. Nissen, B. Norris, A. Ornstein, M. Perkins, S. Plafker, A. Prakash, J. Prussing, W. Reenstra, J. Reid, B. Rhodes, K. Rosato, R. Roth, F. Rubin, T. Sauke, M. Seidel, I. Shalom, J. Shapiro, E. Sheldon, T. Sico S. Silberberg, M. Silfen, T. Sim, S. Sperry, E. Staples, J. Steele, M. Strauss, W. Sun, T. Terwilliger, A. Trikha, A. Ucko, L. Wagner, K. Yeh, and Y. Zussman.

## Proposer's Solution to Speed Problem

Typewriter.

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Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to [gottlieb@nyu.edu](mailto:gottlieb@nyu.edu). For other solutions and back issues, visit the Puzzle Corner website at [cs.nyu.edu/~gottlieb/tr](http://cs.nyu.edu/~gottlieb/tr).