As I write I have the pleasure of listening to my beautiful wife, Alice, playing the cello. She began lessons about two years ago and now takes the instrument very seriously. Perhaps her biggest fan is our golden retriever, Charlie, who always goes to the practice room when Alice is playing. Charlie especially likes the high registers, and when she plays them, he "sings along." We are thinking of a CD entitled *Bach Suites for Cello and Howler*.

This column is dedicated to the victims of the tragic Boston Marathon bombing.

Problems

J/A 1. David Porter, having been dealt five spades, was pleased to hear his partner open with 1 no-trump and his opponents remain silent. Porter knows that this means his partner's distribution was either 4-3-3-3, 4-4-3-3, 5-3-3-2, or 5-4-2-2. What are the probabilities that the partner has exactly two spades, three spades, four spades, or five spades?

J/A 2. Ermanno Signorelli has one for all you wordsmiths and Scrabble players. Assemble a list of English words, using the smallest total number of letters, such that the alphabet is contained, in reverse alphabetical order in the letters of the words. As an example, Signorelli offers aZYgos, ... ItcHinG, FED, and CaBAl.

J/A 3. Phil Lally reports that Jack and Jill, after the episode with the hill, decided to go shopping. They purchased several items each costing a whole number of dollars. Their total expenditure was $207. One of the items cost $1; the others had dollar prices that are prime numbers. The sum of the digits of the price of one item was 7. Furthermore, when the prices are written down, each of the digits 1 through 9 is used exactly once. What were the prices?

Speed Department

John Astolfi believes that instead of considering the expansion of pi to be 3.14159 ... , we should do it in base 2. If we follow this suggestion, does the expansion have: A.) more 0s than 1s; B.) more 1s than 0s; C.) as may 1s as 0s; or D.) impossible to tell.

Solutions

J/F 2. Oops. Somehow communication failed between New York and Cambridge and the text accompanying Barrows's diagram did not appear. We have scouts scouring Connecticut looking for it, but in the interim have placed the full solution on the Puzzle Corner website (cs.nyu.edu/~gottlieb.tr). We apologize for the error.

M/A 1. Larry Kells wants you to consider bridge hands for all four players with South having a 4-3-3-3 distribution and losing all 13 tricks when declarer at no-trump, assuming best play on all sides. What is the most high-card points South can have?

Several readers found 26-point solutions, all of which are basically the same. The following contribution from Mark Bolotin includes an argument that 26 points is the maximum possible.

South can have 26 points as follows. It does not matter how the remaining cards are distributed between North and East.

West	South	East
♠ A K Q 8 7 6 5 4 3 2		♠ A 10 9
♥ 10		
♦ 2		
♣ J		
	♠ J 10 9	
	♥ A K Q J	
	♦ K Q J	
	♣ A K Q	

West cashes his 10 spade tricks from the top to leave a three-card ending. West has his three singletons; East has the three indicated cards. North doesn't have any relevant cards. What three cards must South hold? He has to hold onto two diamonds; otherwise West leads his diamond and East takes the rest. That means South no longer has room for both a heart and a club. West cashes whichever card is now a winner and squeezes South again.

Why is this the most possible points?

A running suit by West requires a suit headed by AKQ, since South has at least a tripleton. Then East-West need either another running suit or an entry to East's hand. Giving East an A to guarantee an entry leads to the above scenario.

For West to be able to run 13 tricks in his own hand, he must have at least 18 points (two suits headed by AKQ). That leaves only 22 points for the other three players.

A suit that runs with one finesse through South requires a suit headed by AKJ and an entry to East's hand. Then either West needs a second suit to run or East needs another entry. With one suit headed by a Q and another headed by QJ, South is limited to at most 24 points

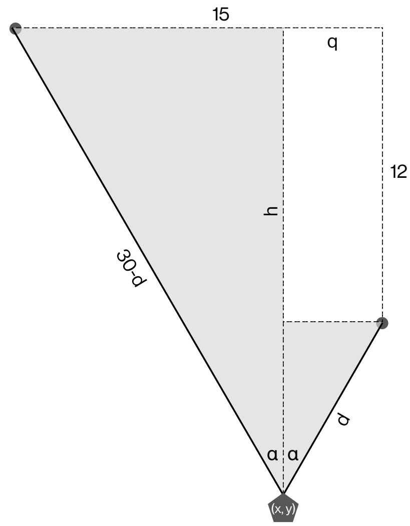
A suit that runs with a repeated finesse through South requires that the suit is headed by AQ and that East has J10 and an entry. Then either West needs a second suit to run or East needs another entry. However, that second entry can only get them to at most 10 running tricks, and East-West have to run 11 to get into a double squeeze position. Even before you find that one more trick, South is limited to at most 25 points (23 by the time you find the needed trick).

A suit that runs with two finesses through South requires a suit headed by AQ10. Two certain entries to East's hand requires at least AK. In addition, either East needs a third entry or West needs another suit to run. With one suit headed by KJ and another headed by J, South is limited to at most 24 points.

M/A 2. Bob Olness offers a problem that can arise when hanging pictures.

A medallion hangs from a 30-centimeter (weightless, frictionless, etc.) string that is attached asymmetrically to the wall, one end at $(x, y) = (0, 0)$, the other end at $(15, -12)$ (coördinates are in centimeters). With the medallion at its equilibrium position, the string will form a lopsided V. Find the (x, y) coördinates of the point on the string from which the medallion hangs.

The following solution is from JD Smith.



$$\sin \alpha = \frac{q}{d} = \frac{15-q}{30-d} \rightarrow \frac{q}{d} = \frac{1}{2} \rightarrow \alpha = 30^\circ$$

$$\cos \alpha = \frac{h}{30-d} = \frac{h-12}{d} = \frac{\sqrt{3}}{2}$$

$$h = \frac{\sqrt{3}}{2}(30-d) = 12 + \frac{\sqrt{3}}{2}d \rightarrow h = 6 + \frac{15\sqrt{3}}{2}$$

$$d = 15 - \frac{12}{\sqrt{3}} \rightarrow q = \frac{d}{2} = \frac{15}{2} = \frac{6}{\sqrt{3}}$$

$$x = 15 - q = \frac{15}{2} + \frac{6}{\sqrt{3}}$$

$$(x, y) = \left(\frac{15}{2} + \frac{6}{\sqrt{3}}, -6 - \frac{15\sqrt{3}}{2}\right) \simeq (10.96, -18.99)$$

The key realization is that since the tension on the string is constant along its length, the two angles from the vertical formed at the medallion must be equal to balance the horizontal forces. This creates two similar triangles (shaded), with corresponding ratios of sides to hypotenuse. The ratio of the horizontal sides reveals the half angle (α=30°, independent of the distance of the second attachment point vertically below the first!). With this angle, the ratio of the vertical sides then gives the coördinates of the medallion: (10.96, –18.99). The result was verified experimentally with string and thumbtacks to within one centimeter (frictionless string being hard to come by in my household).

M/A 3. Howard Cohen has plenty of AND and OR gates but just two inverters. How can he invert three signals *A*, *B*, and *C*? More generally, he wonders if the ratio I/S can ever be less than 2/3, where *I* is the number of inverters and *S* is the number of signals to invert (once again, unlimited AND and OR gates are available).

Larry Stabile's beautiful diagram is on the Puzzle Corner website, cs.nyu.edu/˜gottlieb/tr. Meanwhile, Eric Jensen provides a solution with two parts. In part one, he shows us how to invert three signals with two inverters. This is clever. In part two, he shows us how, for any *S*, to invert *S* signals with two (yes, just *two*) inverters. This part blew me away, and I went into every unlocked office on the floor to commandeer the blackboard and share it. Jensen writes that he first met this problem about 40 years ago.

For part one, Jensen begins this way. I use juxtaposition for AND, a vertical stroke for OR, and ˜ for NOT.

$P = ABC$ (tells us when all 3 signals are true)
$Q = (AB)|(AC)|(BC)$ (when 2 or 3 of 3 are true)
$R = A|B|C$ (1, 2, or 3 of 3)
$S = \tilde{}Q$ (0 or 1 of 3; uses the first inverter)
$T = SR$ (1 of 3)
$U = P|T$ (1 or 3 of 3)
$V = \tilde{}U$ (0 or 2 of 3;, uses the second inverter)
$W = SV$ (0 of 3)
$X = RV$ (2 of 3)

With these building blocks, ˜*A* becomes $W\,|\,(T(B|C))\,|\,X(BC)$, where the first term is for none true, the second for one true but it is one of the others, and the third for two true but they are both of the others. Analogous formulas hold for ˜*B* and ˜*C*.

Now for the real magic, inverting *N* with two inverters. (This wording is mine; Jensen was considerably more scholarly.) Build one of these circuit boards. Ignoring the circuit, the board itself acts as three inverters (it has three inputs *A*, *B*, *C* and three outputs ˜*A*, ˜*B*, ˜*C*). We say the board implements three logical inverters.

Now build a second identical board. Together the boards invert six signals using four inverters. Remove the two physical inverters from the second board; now it doesn't work. But you can revive the second board by using two of the three logical inverters from the first board. This results in two boards acting as four logical inverters, but containing only two physical inverters.

Keep building boards with their physical inverters removed. Each time you add one of these boards, you use two previously available logical inverters but supply three new logical inverters. Thus *N* boards (only one with physical inverters) can invert $N+2$ signals with just two inverters, as desired.

Other Responders

Responses have also been received from F. Albisu, T. Barrows, D. Berg, P. Cassady, J. Feil, R. Giovanniello, D. Goldfarb, J. Hardis, T. Harriman, D. Hoyle, L. Iori, S. Kanter, J. Kotelly, R. Krawitz, K. Lebensold, W. Lemnios, G. Mamon, P. Manglis, Z. Mester, T. Mita, F. Model, A. Muenz, S. Nason, E. Nelson-Melby, J. Prussing, J. Rible, E. Sard, I. Shalom, E. Signorelli, S. Silverberg, J. Steele, S. Strauss, D. Teixeira, S. Toner, S. Ulens, R. Wake, and T. Weiss.

Proposer's Solution to Speed Problem

C; an infinite number of each. If either number were finite, pi would be rational.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.