

Readers who enjoyed Frank Rubin's sumsum puzzle might wish to visit sumsumpuzzle.com and should also be on the lookout for his "Donut Sudoku" book, which should be published soon.

Problems

M/A 1. Larry Kells wants you to consider bridge hands for all four players with the requirements that South has a 4-3-3-3 distribution and loses all 13 tricks when declarer at no trump, assuming best play on all sides. What is the most high-card points South can have?

M/A 2. Bob Olness offers a problem that can arise when hanging pictures.

A medallion hangs from a 30-centimeter (weightless, frictionless, etc.) string that is attached asymmetrically to the wall, one end at $(x, y) = (0, 0)$, the other end at $(15, -12)$ (coordinates are in centimeters). With the medallion at its equilibrium position, the string will form a lopsided V. Find the (x, y) coordinates of the point on the string from which the medallion hangs.

M/A 3. Howard Cohen has plenty of AND and OR gates but just two inverters. How can he invert three signals a , b , and c ?

More generally, he wonders if the ratio I/S can ever be less than $2/3$, where I is the number of inverters and S is the number of signals to invert (once again, unlimited AND and OR gates are available).

Speed Department

Phillip Cassady's team A entered a weird tournament and must play three matches in total against a strong team B and a weaker C. To win the tournament, A must win two games in a row. Phillip can choose to face, in order, either BCB or CBC. Which should he choose?

Solutions

N/D 1. Sorab Vatcha would like you to construct a 9-by-9 sudoku puzzle with the fewest seed numbers required to yield a unique solution, and then solve it.

To paraphrase my former NYU colleague Mal Kalos, this problem (along with its solution) is "well known to those who know it well." Several readers reported sites that give minimal 17-clue solutions. For example Jerrold Grossman refers us to school.maths.uwa.edu.au/~gordon/sudokumin.php. About a year ago a proof was given that 17 clues is the minimum, and

Grossman and Ken Zeger refer us to an *MIT Technology Review* blog post on the subject, technologyreview.com/view/426554/mathematicians-solve-minimum-sudoku-problem. Bob Donaldson recommends a Google search on "there is no 16 clue sudoku" to find the proof and other interesting Web pages. Frank Rubin and Richard Hess sent us initial and final configurations. Rubin's is reprinted below.

1	2	0	3	4	0	0	0	0
3	0	0	0	0	0	5	0	0
0	0	0	0	0	0	0	6	0
0	0	0	0	0	0	3	0	1
0	0	6	0	0	7	0	0	0
0	0	8	0	0	0	0	0	0
0	0	7	0	0	8	0	9	0
4	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0

1	2	5	3	4	6	9	8	7
3	6	4	7	8	9	5	1	2
8	7	9	2	5	1	4	6	3
9	4	2	8	6	5	3	7	1
5	3	6	1	2	7	8	4	9
7	1	8	4	9	3	6	2	5
2	5	7	6	3	8	1	9	4
4	8	3	9	1	2	7	5	6
6	9	1	5	7	4	2	3	8

N/D 2. Steve Silberberg views life not as a bowl of cherries but, instead, as a four-dimensional integer lattice. We are located at the origin and wish to get to (x, y, z, t) with all four positive integers. Each individual move is to advance one unit *forward* in one dimension. How many different paths are possible as a function of the target (x, y, z, t) ? For example, two of the legal paths to $(1, 1, 1, 1)$ would be

$(0, 0, 0, 0) \rightarrow (0, 0, 1, 0) \rightarrow (0, 1, 1, 0) \rightarrow (0, 1, 1, 1) \rightarrow (1, 1, 1, 1)$

and

$(0, 0, 0, 0) \rightarrow (0, 0, 0, 1) \rightarrow (1, 0, 0, 1) \rightarrow (1, 0, 1, 1) \rightarrow (1, 1, 1, 1)$

Mark Perkins reports that when he first looked at this problem, he didn't really have a good idea about how to proceed, but then he realized it was a simple combinatorial problem. The answer is

$$\frac{(x + y + z + t)!}{(x!)(y!)(z!)(t!)}$$

Perkins's first key observation was that the lattice problem is equivalent to the following counting problem. Given x red balls, y yellow balls, z green balls, and t blue balls, in how many different orders can the balls be placed (noting that identically colored balls are indistinguishable)?

The second key is to note that since all steps are in the positive direction, there is a one-to-one mapping between legal paths and orderings of the balls. For example, a red ball represents a step in the x direction. The above formula is the well-known solution to the counting problem with identical subclasses.

Basically, the numerator is the number of orderings considering each ball to be distinguishable. Each term in the denominator is the number of orderings of balls of a given color that are to be discounted (i.e., considered the same) because balls of a given color are indistinguishable from each other.

Willy Burke notes that the number of paths grows very quickly. The solutions for (1,1,1,1), (4,4,4,4), and (10,10,10,10) are, respectively, 24; 63,063,000; and 4,705,360,871,073,570,000,000.

N/D 3. Nob Yoshigahara, honoring the late Japanese cryptogramist Kyoko Ohnishi, offers one of her masterpieces. As usual, each letter stands for a unique digit.

$$\begin{array}{r} \text{L O V E} \\ \times \quad \text{I S} \\ \hline \square\square\square\square \\ \square\square\square \\ \hline \text{B L I N D} \end{array}$$

As far as I can see, everyone used a computer search to find the solution; some solvers guided the search more than others. Richard Blair writes:

"The apparent simplicity of N/D3 in the recent issue captivated me. I haven't a clue how to solve it deductively, but as an amateur BASIC programmer, I figured the computer could find the solution(s) for me.

"My Tandy CoCo3 took 17+ hours to find six solutions. Four of these have leading zeros in one of their factors, which might not be considered kosher [*it is not —ed.*]. The remaining two

are okay on that score, but one of them generates only four digits for the first partial product, and the statement of the problem shows five digits there. So the only unassailable solution is $1,572 \times 39 = 61,308$.

"After finding the solution(s), I was struck by a mild attack of 'programmer's disease' [*we use other terminology when discussing these techniques in compiler construction courses —ed.*], and spent some time making the BASIC program more efficient."

The following schematic from Willy Burke illustrates the valid (top) and invalid configurations.

$\begin{array}{r} 1572 \\ \times 39 \\ \hline 14148 \\ 4716 \\ \hline 61308 \end{array}$	→	$\begin{array}{r} \text{L O V E} \\ \times \quad \text{I S} \\ \hline \text{L} \times \text{L} \times \text{D} \\ \times \text{V L B} \\ \hline \text{B L I N D} \end{array}$
$\begin{array}{r} 3598 \\ \times 12 \\ \hline 7196 \\ 3598 \\ \hline 43176 \end{array}$	→	$\begin{array}{r} \text{L O V E} \\ \times \quad \text{I S} \\ \hline \text{N I V D} \\ \text{L O V E} \\ \hline \text{B L I N D} \end{array}$

Other Responders

Responses have also been received from M. Brill, J. Chandler, T. Chase, B. Cook, J. Harmse, B. Kulp, G. Lauer, P. Lively, H. Marshall, T. Mita, E. Nelson-Melby, A. Ornstein, G. Roberts, R. Schooler, A. Sezginer, E. Sheldon, J. Simmonds, M. Tarsi, and D. Turek.

Proposer's Solution to Speed Problem

BCB—it is better to play the stronger team twice. To win two in a row, Phillip must win the middle game, and he has two chances against the other team. So you want the weaker opponent in the middle game. (This can be confirmed by a careful analysis.)

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.