This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 3) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2012 yearly problem is in the "Solutions" section.

Problems

Y2013. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 3 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 3 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Another bridge problem from the almost infinitely well-named Larry Kells.

Specify North-South cards with the following property, always assuming best play and a fixed-suit contract with South as declarer: the difference between the number of tricks taken with the most favorable opposing distribution and the number taken with the least favorable opposing distribution is maximized.

J/F 2. Philip Cassady has four spheres of radius b resting in contact at the bottom of a spherical bowl of radius a, their four centers being at the corners of a horizontal square. A fifth identical sphere is placed upon them. What conditions on the relationship between a and b are required for each sphere to be in equilibrium under the action of its weight and the reactions of the bowl and the other spheres? Regard all contacts as smooth with no friction.

Speed Department

Peter Mamorek wants to know, "How do you prove literally that 11 + 2 = 12 + 1?"

Solutions

Y2012. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 2 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 2 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign, however, does count as an operator.

I combined the solutions from Michael Piazza and James Simmonds to obtain the following.

$1 = 1^{202}$
$2 = 22^0 + 1$
$3 = 10/2 - 2$
$4 = 2^0 + 1 + 2$
$5 = 2 + 0 + 1 + 2$
$6 = 10 - 2^2$
$7 = 10/2 + 2$
$8 = 20 - 12$
$9 = (2 + 1^0)^2$
$10 = 20^1/2$
$11 = 20/2 + 1$
$12 = 22 - 10$
$13 = 2^0 + 12$
$14 = 10 + 2^2$
$16 = (10 - 2) \times 2$
$17 = 20 - 1 - 2$
$18 = 20^1 - 2$
$19 = 20 + 1 - 2$
$20 = 20 \times 1^2$
$21 = 20 + 1^2$
$22 = 20 \times 1 + 2$
$23 = 20 + 1 + 2$
$24 = 12 \times 2 + 0$
$25 = (10/2)^2$
$32 = 20 + 12$
$38 = (20 - 1) \times 2$
$39 = 20 \times 2 - 1$
$40 = 20 \times 1 \times 2$
$41 = 20 \times 2 + 1$
$42 = (20 + 1) \times 2$
$50 = 10^2/2$
$51 = 102/2$
$60 = 120/2$
$64 = (10 - 2)^2$
$98 = 10^2 - 2$
$100 = 102 - 2$

S/O 1. Larry Kells has another series of minimum-points bridge problems. This time he wants to know the smallest number of (high-card) points a player can have and still be sure of making 7 no-trump. How about 6 no-trump? Three no-trump? One no-trump?

Scott Nason particular enjoyed the 3 no-trump subproblem and writes:

"In order for one player to ensure that they can make 7 no-trump, that player must have all four aces, plus the ability to run nine additional tricks. This can 'best' be done with a suit of AKQxxxxxxx. Thus, with 21 high-card points, 7 no-trump is certain. Interestingly, there is no hand with less than 21 high-card points that is guaranteed to make even 6 no-trump, since without two 'stoppers' in each suit, declarer cannot afford to lose the lead until 12 tricks are won.

"Three no-trump is more complicated and more interesting. Certainly it requires a stopper (even if it is in the fifth round) in every suit, and the ability to take nine tricks as soon as your one stopper is knocked out in any side suit. A A AKQJT98 JT98 will accomplish the task, ensuring nine tricks as soon as the lead is captured, while ensuring that the opponents cannot take five tricks in the meantime. Note that a side stopper with a 10-high suit is possible, but only if the suit is at least five long, which leaves only eight tricks available in the other suits. So the answer is that 3 no-trump requires 'only' 19 high-card points.

"And 1 no-trump is not a lot 'better.' You still need some sort of stopper in every suit, lest they run seven or more tricks in that suit. And without two stoppers in every suit, declarer needs to be prepared to take all seven tricks as soon as the lead is attained. This is possible with 'only' 18 high-card points, as follows: AKQJT9 A T9876 A. This hand is guaranteed to make at least 2 no-trump, but cannot even make 1 if any of the high-card points is eliminated.

"An interesting, and lower-point-count, solution is possible if the two partner hands are combined. Here it is possible to create stoppers or run long suits with slightly fewer high-card points. For example, while 7 no-trump still requires all four aces, the long side suit can be 12 long to the AQJ and still be guaranteed to run. Thus, these two hands will always take 13 tricks: A AQJxxxxxxxxx, with voids in both minors, paired with x void Axxxxx Axxxxx, with only 19 high-card points combined. And the two hands will always take at least 12 tricks if the first hand (with 12 hearts) is dummy and declarer has one ace and the K in the fourth suit, for a total of 18 high-card points. The same dummy will produce at least nine tricks opposite: x void Axxxxx T987xx, a total of 15 high-card points. And it will produce at least seven tricks opposite: x void JT9876 JT987, a total of 13 high-card points."

S/O 2. Ermanno Signorelli sent us this problem he read in Marilyn vos Savant's "Ask Marilyn" column on Parade.com. The original proposer, A. Wright, has seen 70 cows take 24 days to eat all the grass in a pasture. Other times this same pasture would be denuded by 30 cows in 60 days. Wright, Savant, and Signorelli want to know how many cows would be supported for 96 days. You should assume that in all cases the grass starts at the same height and grows at the same rate.

Marlon Weiss solves this by introducing a new quantity of grass called a "cowday," the amount eaten by one cow in one day. Marlon writes:

"In the first case, 70 cows in 24 days eat 1,680 cowdays of grass. In the second, 30 cows in 60 days eat 1,800 cowdays of grass. The difference of 120 cowdays is the growth of the grass in the 36 additional days.

"At zero days, the pasture contains $1{,}680 - 120/36 \times 24 = 1{,}600$ cowdays of grass. Over 96 days, the pasture would have $1{,}600 + 120/36 \times 96 = 1{,}920$ cowdays of grass. Hence $1{,}920/96 = 20$ cows could be supported for 96 days."

S/O 3. David Shin, who has *many* friends, knows an infinite number of wizards. David told them to prepare for the following contest and see if they can derive a method that guarantees a winning probability of at least 90 percent.

Each wizard will be assigned a random hat, either black or white, with a probability of 1/2 for each choice. The wizards can see everyone's hat except for their own. At the count of three, each wizard must either guess the color of his or her hat or abstain from guessing. The wizards collectively win if, among them, there are an infinite number of correct guesses and zero wrong guesses.

This one requires considerable cleverness. It is apparently well known (to some, not including me) that this can be done for $n = 2^k - 1$ wizards, and the extra trick here is extending it to infinitely many wizards. One indication of the subtlety is the following remark from the proposer:

"You might be thinking, 'How is this even possible?' Each individual guess will be wrong 50 percent of the time. How could the entire group do better than 50 percent?

"The answer to this paradox lies in the power of abstaining. It is true that each individual guess will be wrong 50 percent of the time. However, not each wizard has to guess! If we define 'failing' as guessing incorrectly, each wizard can avoid failing with high probability by abstaining often.

"You may notice in the $n = 3$ solution (and also the $n = 2^k - 1$ solution) that when the committee is right, they only have one correct guess, while when they are wrong, they are wrong in spectacular fashion: every wizard guesses incorrectly. There are an equal number of right and wrong guesses when summed over all possible hat distributions, but the idea is to pool wrong guesses together and spread out right guesses. The infinite-wizards case simply uses this idea to an infinite degree."

Because of space limitations I have placed four solutions, from Mark Fischler, Jerrold Grossman, John Klincewicz and Herb Shulman, and the proposer, on the Puzzle Corner website (cs.nyu.edu/~gottlieb/tr).

Better Late Than Never

2012 S/O SD. Several readers noted that the author's answer (1/2) was correct but my added explanation had a typo. The area is 1/2 base times height.

Other Responders

Responses have also been received from F. Albisu, D. Aucamp, R. Bator, B. Beachkofski, S. Berger, R. Bird, M. Brill, A. Cetinbudaklar, R. and A. Craig, R. Currier, C. Dale, D. Diamond, J. Feil, J. Freilich, E. Friedman, J.-P. Garric, R. Giovanniello, H. Goldman, D. Goldstone, J. Hardis, J. Harmse, H. Hyman, H. Ingraham, L. Kahn, M. Kenworthy, P. Kramer, B. Kulp, M. Langeveld, W. Lemnios, R. Lipes, S. Nason, E. Nelson-Melby, A. Ornstein, M. Perkins, J. Prussing, E. Sard, P. Schottler, I. Shalom, E. Signorelli, J. Simmonds, A. Sood, S. Sperry, J. Steele, M. Strauss, M. Thompson, T. Threadgold, S. Vatcha, D. Wachsman, and B. Wright

Proposer's Solution to Speed Problem

There are 2 "literal," as in letter for letter, solutions. Either move an I in "XI + II" or rearrange a bunch of letters in "ELEVEN + TWO."

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.