Readers who have worked on our yearly problem might enjoy the "Triple Nine Society Wall Clock" that Warren Smith found at www.flickr.com/photos/lwr/1378672867.

Since this is the first issue of a new academic year, let me once again review the ground rules. In each issue I present three regular problems, the first of which is normally related to bridge (or chess or some other game), and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e., four months) later, one submitted solution is printed for each; I also list other readers who responded. For example, the current issue contains solutions to the regular problems posed in May/June.

The solutions to the problems in this issue will appear in the January/February column, which I will need to submit in mid-October. Please try to send your solutions early to ensure that they arrive before my deadline. If you send e-mail with attached solutions, please include your name on the attachment as well as the e-mail. Sometimes I print the attachment and it gets separated from the message.

Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given on the facing page. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that have remained unsolved.

PROBLEMS

S/O 1. Larry Kells has another series of minimum-points bridge problems. This time he wants to know the smallest number of (high card) points a player can have and still be sure of making 7 no-trump. How about 6 no-trump? Three no-trump? One no-trump?

S/O 2. Ermanno Signorelli sent us this problem he read in Marilyn vos Savant's "Ask Marilyn" column on Parade.com. The original proposer, A. Wright, has seen 70 cows take 24 days to eat all the grass in a pasture. Other times this same pasture would be denuded by 30 cows in 60 days. Wright, Savant, and Signorelli want to know how many cows would be supported for 96 days. You should assume that in all cases the grass starts at the same height and grows at the same rate.

S/O 3. David Shin, who has *many* friends, knows an infinite number of wizards. David told them to prepare for the following contest and see if they can derive a method that guarantees a winning probability of at least 90 percent.

Each wizard will be assigned a random hat, either black or white, with a probability of 1/2 for each choice. The wizards can see everyone's hat except for their own. At the count of three, each wizard must either guess the color of his or her hat or abstain from guessing. The wizards collectively win if, among them, there are an infinite number of correct guesses and zero wrong guesses.

SPEED DEPARTMENT

Given a unit circle, what is the largest possible area of a triangle ABC with A at the center of the circle and B and C on the circle?

SOLUTIONS

M/J 1. Dan Karlan offers another sudoku-based problem.

In sudoku parlance, a box is one of the nine three-by-three subarrays of cells within which each digit must appear exactly once. Within each box, a "triplet" is one of the six three-digit sequences, three horizontal and three vertical. Thus, there are 54 triplets in the entire game. A "consecutive triplet" is one in which the three digits are consecutive integers, in any order—for example, 1-2-3 or 7-8-6. The diagram at right shows four consecutive triplets.

8	3	5	1	7	2	6	4	9
1	6	7	3	9	4	5	2	8
9	4	2	8	6	5	7	1	3
2	8	6	9	3	7	1	5	4
7	5	1	4	8	6	3	9	2
3	9	4	2	5	1	8	7	6
4	7	3	6	1	9	2	8	5
6	1	9	5	2	8	4	3	7
5	2	8	7	4	3	9	6	1

It is easy to see that the maximum number of consecutive triplets that can occur in a box is three, and you can construct a legal sudoku puzzle with 27 consecutive triplets, the maximum number possible.

What is the minimum number of consecutive triplets? The diagram shows it is at most four.

Robert Wake writes that the same elementary sudoku that proves you can have 27 consecutive triplets also proves that you can have none at all.

123 456 789 231 564 897 312 645 978
456 789 123 564 897 231 645 978 312
789 123 456 897 231 564 978 312 645

This construction generalizes to show that given any valid sudoku box, there exists a full sudoku with the same set of six triplets in every box. Hence we need only find a box with no consecutive triplets (in rows, columns, or even diagonals). The diagram in the original problem had several such boxes, but Wake points out that if we had to do it from scratch, the easiest way would be to permute one of the diagonals in the above example.

523 496 781 235 964 817 352 649 178
496 781 523 964 817 235 649 178 352
781 523 496 817 235 964 178 352 649

M/J 2. When Dushan Mitrovich comes home he empties his pocket of coins into an enormous coffee can. Each day he is equally likely to have one cent, two cents, three cents, ... 99 cents.

For some reason, the distribution in his pocket always maximizes the number of large coins, given the total value of the coins. For example, if he has 27 cents it will always be one quarter and two pennies. What is the likely distribution of coins in the can after many days?

My wording of the problem was unfortunately ambiguous in two ways. First, "large" was a poor choice. Are dimes larger than nickels or smaller than pennies? We intended the former. Second, are 50-cent pieces permitted? Several responders, including Aaron Ucko, supplied answers both with and without half-dollars. Ucko writes:

"In general, the answer depends somewhat on whether [Mitrovich] has half-dollars when appropriate. If he does, he will have one 50/99 of the time (when his change falls between 50 and 99 cents inclusive) and none the rest of the time, and likewise for quarters (albeit with intervals of [25,49] and [75,99]); otherwise, he will have one quarter over [25,49], two over [50,74], and three over [75,99] for an expected count of 150/99 = 50/33 on any given day.

"On the other end of the numismatic range, pennies have a simple pattern: none for multiples of five cents, one for totals congruent to one modulo five, and so on up to four for totals congruent to four modulo five, for an expected count of $20 \times (1 + 2 + 3 + 4) / 99 = 200/99$. Intermediate coins' statistics are likewise periodic: over the representative interval [25,49], we have one nickel on [30,34], one dime on [35,39], one of each on [40,44], and two dimes on [45,49] for a total of 10 nickels and 20 dimes; scaling up to cover the full ange of possible totals gives on average 40/99 nickels and 80/99 dimes.

"Thus, after n days (for large values of n), I would expect $200n/99$ pennies, $40n/99$ nickels, $80n/99$ dimes, and either $50n/33$ quarters or $50n/99$ quarters and $50n/99$ half-dollars. Regardless, these coins add up to $4{,}950n/99 = 50n$ cents, as well they should."

M/J 3. Neil Cohen found a Sunday *Times* article about Freeman Dyson that contained the following problem. Dyson solved it very quickly, but others needed much more time.

Is there a positive integer with the property that if you move its rightmost digit all the way to the left (for example, 112 becomes 211), the result is exactly double the original value?

Deniz Ertaz enjoyed this and writes: "Very clever problem. Let the original (n-digit) number be $10A+B$, where A and B are integers such that $10^{n-2} \le A \le 10^{n-1}$ and $0 \le B < 10$. We are looking for solutions to the integer identity $2 \times (10A + B) = 10^{n-1}B + A$.

"Rearranging, we get $19A = (10^{n-1} - 2)B$. Since 19 is prime and $B < 10$, $10^{n-1} - 2$ must be divisible by 19, with the quotient A/B. To find the solutions, do a long division of 1/19 and continue the division until you reach a remainder of 2. This happens whenever n is a multiple of 18. For each such n, there are eight solutions, corresponding to $2 \le B < 9$. The smallest solution is 105263157894736842 ($n = 18$, $B = 2$). See the table for the first two series of solutions. Notice that numbers within each series are circular shifts of each other, and the solutions for $n = 18m$ can be obtained by concatenating the $n = 18$ solution m times!"

B	$n = 18$	$n = 36$
2	105263157894736842	105263157894736842105263157894736842
3	157894736842105263	157894736842105263157894736842105263
4	210526315789473684	210526315789473684210526315789473684
5	263157894736842105	263157894736842105263157894736842105
6	315789473684210526	315789473684210526315789473684210526
7	368421052631578947	368421052631578947368421052631578947
8	421052631578947368	421052631578947368421052631578947368
9	473684210526315789	473684210526315789473684210526315789

Generalizations from Chatchawin Charoen-Rajapark and Frank Marcoline can be found on the Puzzle Corner website. John Prussing believes that base 10 was not required and notes that 10 is twice 01 in binary. (Our attorneys will decide if my giving an example using the digit 2 precludes base 2 solutions.)

BETTER LATE THAN NEVER

2011 N/D 3. Dan Sidney has found a class of shapes that can be filled in finite time. His report is on the website.

2012 J/F 2. Bob Currier, Aaron Ucko, Scott Smith, and Robert Wake disagree with the published solution. Analyses from Wake and Smith appear on the website.

OTHER RESPONDERS

Responses have also been received from F. Albisu, A. Andersson, G. Benton, R. Byard, C. Carbone, M. Chartier, C. Connell, C. Dailey, J. Desmond, D. Diamond, B. Edelman, R. Giovanniello, M. Gordon, P. Groot, J. Grossman, T. Hafer, E. Hestermann, Y. Hinuma, I. Lai, W. Lemnios, J. Licini, R. Lipes, C. Lu-Fong, T. Maloney, J. Martin, T. Mita, C. Monroe, E. Nelson-Melby, J. Norvik, A. Ornstein, C. Perera, K. Price, B. Rhodes, S. Richmond, K. Rosato, J. Russell, E. Sard, S. Schulman, A. Sezginer, I. Shalom, E. Sheldon, A. Shulman, E. Signorelli, A. Smith, E. Staples, G. Stith, and S. Vatcha.

PROPOSER'S SOLUTION TO SPEED PROBLEM

1/2. The area—base times height—is maximized when AB is perpendicular to AC. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.