What a wonderful late winter and early spring here in the Northeast. We visited Rome in February only to encounter their heaviest snow in 25 years (this is measured in inches—really, centimeters—not feet or meters) and returned to warm, sunny days. We subsequently twice visited our son David and his lovely wife, Mari, in San Diego, which they proclaim (with some justification) is the home of permanently wonderful weather. Both times were damp and cool, and in each case we left and returned to warm, sunny days. In fact, during one week of glorious New York weather, I commented to our dog, Charlie, when walking him in the morning, "Another beautiful day—how boring. This must be how David feels."

PROBLEMS

J/A 1. Tom Hafler has created a three-dimensional sudoku-like problem he calls "wordoku," involving four horizontal levels. His instructions are as follows.

"This is a 4x4x4 cube with 16 different symbols. It is solved exactly like a normal two-dimensional sudoku problem except that each symbol can occur only once in each cardinal plane, including the vertical dimension. Also, on this one a bit of intuition is required. When the solution complete, a message or pattern will emerge."

U		L	
M		T	
	N		W
	S		E

I		4	
	K		O
	Y		A
Z		P	

K		O	
Y		A	
	P		Z
	4		I

J/A 2. Warren Smith's interest in improved voting schemes for democracies extends to puzzles on voting and related topics, including the following.

Suppose there are four dice: Blue, Green, Red, and White. These dice have different numbers than usual printed on their six faces. After observing a long sequence of experiments rolling pairs of these dice, you conclude the following:

• When both are rolled simultaneously, the blue die gives a higher number than the green die two thirds of the time.

• When both are rolled simultaneously, the green die gives a higher number than the red die two thirds of the time.

• When both are rolled simultaneously, the red die gives a higher number than the white die two thirds of the time.

You are now asked to consider rolling the blue and white dice simultaneously. What can you conclude about the probability P that the blue die will produce a higher value than the white one?

J/A 3. Victor Barocas teaches a class in which students sit in rows. When he gives an exam, he asks students not to leave early unless they can do so without having to disturb anyone else—that is, unless there is an unobstructed path to a side of the room. As a result, a bunch of students sometimes build up and are then "released" when a student on one end finishes. Obviously, when the second-to-last student in a given row finishes, everyone in that row can leave except for the last student, because the row has two ends. If the row has N students in it, what is the expected number of students to leave when the second-to-last student finishes?

SPEED DEPARTMENT

Edwin Rosenberg offers us a problem that might be a little hard for speed but does have a one-line answer. He asks, "What property is shared by only the four following numbers: 403, 1,207, 2,701, and 76,63?"

SOLUTIONS

M/A 1. All the responders agree that the key first move is to sacrifice the rook, and there are then many variations. The point is that with the large material advantage, Black needs only to weather the current storm. The following solution is from Mark Moss.

White threatens checkmate on g7. A defense that is too blunt will lose immediately:

1. ... g6
2. Qh6 any move
3. Qg7 mate

A strong defensive formation for Black would be to get the pawns to f7, g6, and h7, and to get the Black bishop to f8 to defend the g7 square. The rook sacrifice gives Black a key tempo.

1. ... Rg4
2. Qg4 g6

There is no better alternative for White than taking the rook, which gives Black the time needed to defend.

Here are some possible continuations:

3. Qg5 Re8
4. Qh6 Re1 mate

3. Qg5 Re8
4. Bc3 Nb5 threatening to drive the bishop from the a1-h8 diagonal

3. Qg5 Re8
4. a4 Bf8

Other alternatives involve moving the queen to h4, from where it protects the e1 square against the upcoming Re8 mate threat by Black and still allows the White queen to reach h6 and f6 in later variations:

3. Qh4 Re8
4. a4 Bf8
5. Bc3 (threatening Qf6 with mate on g7 or h8)
... Bg7

3. Qh4 Re8
4. Bc3 Be5 blocking the a1-h8 diagonal

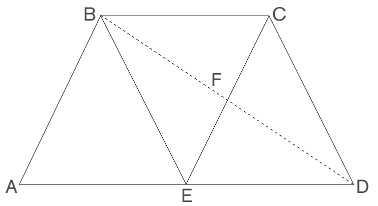
Even the "sloppiest" defense still stops the immediate mate threat and leaves Black with an overwhelming advantage:

3. Qg5 Re8
4. a4 Be5
5. Be5 Re5
6. Qe5 Nc6

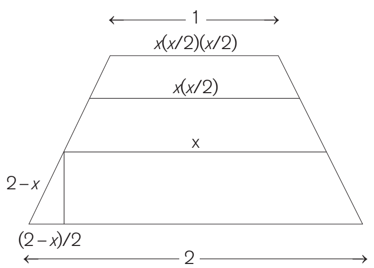
This variation forces the issue defensively: the white bishop is exchanged or forced from the a1-h8 diagonal, ending any practical mating threats by White. This satisfies the "anti-checkmate" stipulation and still leaves Black with an overwhelming material advantage that should win.

M/A 2. The following solution is from Eugene Sard. The solution for part 1 is simply the three equilateral triangles in the problem's trapezoidal figure.

The solution for part 2 is shown in the diagram below, with three 30-60-90 triangles drawn by connecting the lower right corner of the trapezoid and its upper left corner. (Since BCDE is a rhombus, its diagonals are perpendicular. Hence triangles BFC, DFC, and ABD are all 30-60-90; the first two are the same size and the third is different.)



The solution to part 3 is shown in the diagram below, with the original trapezoid divided by two horizontal lines. We assume the equilateral triangles have side length 1, implying that the bottom of the trapezoid has length 2. The resultant three similar trapezoids have equal corresponding interior angles, and the location of the lines is determined by solving for the common ratio of corresponding sides of the component trapezoids. Let x be the length of the top side of the bottom trapezoid, which is also the bottom side of



the middle trapezoid. Then, since the bottom side of the bottom trapezoid (which is also the bottom side of the original trapezoid) is 2, the common ratio of sides is $x/2$. The top side of the middle trapezoid, which is also the bottom side of the upper trapezoid, then equals $x(x/2)$, and the top side of the top trapezoid equals $x(x/2)(x/2) = x^3/4$. Since the top side of the top trapezoid is the top side of the original trapezoid, its length is 1 and $x = 4^{1/3} \approx 1.6$.

The location of the end points of the horizontal lines are determined as shown on the figure below for the bottom trapezoid, using the fact that the hypotenuse of a 30-60-90 triangle is twice the side opposite the 30° angle, which shows that the left (and right) side of the bottom trapezoid is equal to the difference between its bottom and top sides, or $2 - x = 2 - 4^{1/3}$. Similarly, the sides of the middle trapezoid are $x - x(x/2)$, and the sides of the top trapezoid are $x(x/2) - 1$.

M/A 3. Jerome Licini enjoyed this problem and writes:

$1{,}712 \times 303 = 518{,}736$

$907 + 1 = \sqrt{824{,}463 + 1}$

$3{,}073 \times 3{,}073 + 3{,}073 + 3{,}073 + 3{,}073 + 3{,}073 = 9{,}455{,}621$

I am impressed by how ingenious these puzzles were.

BETTER LATE THAN NEVER

2011 N/D 3. Bob Ackerberg notes that we missed a dot over $h(t)$ in the fourth equation from the bottom of the left column. Dan Sidney believes there are some bowl shapes that can fill in finite time. For example, he asserts that a bowl (solid of revolution) with the "seagull" shape cross-section fills in finite time. His full response is on the Puzzle Corner website, cs.nyu.edu:˜gottlieb/tr.

2012 J/F 2. Robert Wake found a 20-card hand with no "set."

2012 M/A SD. Naomi Markovitz believes that "a 'more immediate' explanation is that the common point of the medians of a triangle splits each median into two segments of ratio 2:1 where the longer segment is the one with a vertex as an endpoint." Bob Weggel feels it is "more obvious [to connect] the midpoints of the sides of the triangle, dividing it into four identical smaller triangles." He adds, "The circle that inscribes the big triangle also circumscribes the center one of the four little ones; there the area ratio is 1:4."

OTHER RESPONDERS

Responses have also been received from M. Burba, S. Feldman, E. Friedman, R. Giovanniello, A. Javaheri, E. Nelson-Melby, A. Ornstein, B. Rhodes, and S. Vatcha.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Each is the product of a palindromic pair of two-digit primes. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.