

It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a large queue of regular problems and a comfortable supply of bridge and speed problems.

**PROBLEMS**

**M/J 1.** Dan Karlan offers another sudoku-based problem.

In sudoku parlance, a box is one of the nine three-by-three sub-arrays of cells within which each digit must appear exactly once. Within each box, a “triplet” is one of the six three-digit sequences, three horizontal and three vertical. Thus, there are 54 triplets in the entire game. A “consecutive triplet” is one in which the three digits are consecutive integers in any order—for example, 1-2-3 or 7-8-6. The diagram below shows four consecutive triples.

It is easy to see that the maximum number of consecutive triplets that can occur in a box is three, and you can construct a legal sudoku puzzle with 27 consecutive triples, the maximum number possible.

What is the minimum number of consecutive triples? The diagram shows it is at most four.

8	3	5	1	7	2	6	4	9
1	6	7	3	9	4	5	2	8
9	4	2	8	6	5	7	1	3
2	8	6	9	3	7	1	5	4
7	5	1	4	8	6	3	9	2
3	9	4	2	5	1	8	7	6
4	7	3	6	1	9	2	8	5
6	1	9	5	2	8	4	3	7
5	2	8	7	4	3	9	6	1

**M/J 2.** When Dushan Mitrovich comes home he empties his pocket of coins into an enormous coffee can. Each day he is equally likely to have one cent, two cents, three cents, ... 99 cents.

For some reason, the coin distribution in his pocket always maximizes the number of large coins, given the total value of the coins. For example, if he has 27 cents it will always be one quarter and two pennies.

What is the likely distribution of coins in the can after many days?

**M/J 3.** Neil Cohen found a Sunday *Times* article about Freeman Dyson that contained the following problem. Dyson solved it very quickly, but others needed much more time.

Is there a positive integer with the property that if you move its rightmost digit all the way to the left (for example, 112 becomes 211), the result is exactly double the original value?

**SPEED DEPARTMENT**

Sorab Vatcha would like you to fill in the missing numbers for these two sequences.

- 1, 2, 9, 64, 625, \_\_\_\_\_.
- 7, 16, 29, 46, \_\_\_\_\_, 92.

**SOLUTIONS**

**J/F 1.** Longtime contributor Frank Rubin has created a new class of grid-based arithmetic puzzles and has created a website with expanded rules, hints, etc. ([sumsumpuzzle.com/sumsum.htm](http://sumsumpuzzle.com/sumsum.htm)).

	6	31	22	12	19	15	8	8	12	11
	2	5	14	20	8	21	28	15	8	23
	28			4	9			13	16	2
13, 8, 5										
27, 9										
11, 25										
29, 7		8				4				
13, 5, 18										
26, 5, 5										
8, 18, 10										
8, 5, 23										
16, 20										
1, 18, 17										

The 10-by-10 grid above has a descriptive border column and row on the left and top, respectively. The problem is for you to black out two nonadjacent squares in each column and row and then write the digits from 1 to 8 once each in the eight remaining squares. Each run of adjacent digits must add up to the corresponding sum shown to the left or on top. If neither black square is at the end of the row or column, there are three runs; if one is at the end, there are two. If both are at ends, there would

	6	31	22	12	19	15	8	8	12	11
	2	5	14	20	8	21	28	15	8	23
	28			4	9			13	16	2
13, 8, 15	6	7		5	2	1		8	4	3
27, 9		6	3	7	4	2	5		1	8
11, 25	2	4	5		8	3	1	6	7	
29, 7		8	1	6	5	4	2	3		7
13, 5, 18	3	2	7	1		5		4	8	6
26, 5, 5	4	1	6	8	7		3	2		5
8, 18, 10	5	3		2	1	7	8		6	4
8, 5, 23	8		2	3		6	4	7	5	1
16, 20	7	5	4		3	8	6	1	2	
1, 18, 17	1		8	4	6		7	5	3	2

be just one run, but this situation does not occur in the problem here.

Even a gratuitous typo did not spoil the appeal of this problem. As many readers noticed, “13, 8, 5” should have been “13, 8, 15.”

Once the typo was overcome, there was agreement on the solution. The one shown is from Jonathan Hardis. Several readers enjoyed this problem and went to Frank’s website to find others.

**J/F 2.** Glen Case doesn’t think of tennis or bridge when he achieves a set. Instead, he’s referring to the card game SET, in which each card has four features: shape, color, shading, and number (see [www.setgame.com](http://www.setgame.com).) Each feature can have one of three states. Thus, the deck is made of  $3^4 = 81$  distinct cards. A “set” is a group of exactly three cards for which each feature is either in a single state or in three different states.

Chase wonders, given some number  $N$  of randomly selected SET cards, what is the probability of there being no “set”? For  $N = 1$  and  $N = 2$ , the probability is of course one: three cards are needed for a set. For  $N = 3$  the probability is  $78/79$ : given any two cards there is only one other card that will complete the set.

There is some disagreement on this problem. The proposer believes that  $P(20) > 0$ , whereas Ed Sheldon asserts that for  $N > 14$ ,  $P(N) = 0$ . Sheldon calls any two cards a base pair and notes that only one other card completes a set. He then offers the following table for  $P(N)$ .

$N-1$	Base Pairs	Total Pool	NonSET Pool	$P(N)$	$P(N)$
0	0	81	81	$P(1) = 1$	1
1	0	80	80	$P(2) = 1$	1
2	1	79	78	$P(3) = P(2) * 78/79$	0.9873
3	3	78	75	$P(4) = P(3) * 75/78$	0.9493
4	6	77	71	$P(5) = P(4) * 71/77$	0.8754
5	10	76	66	$P(6) = P(5) * 66/76$	0.7602
6	15	75	60	$P(7) = P(6) * 60/75$	0.6082
7	21	74	53	$P(8) = P(7) * 53/74$	0.4356
8	28	73	45	$P(9) = P(8) * 45/73$	0.2685
9	36	72	36	$P(10) = P(9) * 36/72$	0.1342
10	45	71	26	$P(11) = P(10) * 26/71$	0.04916
11	55	70	15	$P(12) = P(11) * 15/70$	0.01053
12	66	69	3	$P(13) = P(12) * 3/69$	0.0004580
13	78	68	0	$P(14) = P(13) * 0/68$	0

For  $N > 14$ ,  $P(N) = 0$ .

**J/F 3.** Presumably in deference to my “second career” in computer science, Jerry Grossman sends us a “buggy” problem from Laci Babai, who learned it growing up in Hungary.

Maria is terrified of ticks and wants to create a structure to prevent the infinitesimally small tick in her bedroom from crawling on

her while she is asleep. The tick is somewhere on the walls or ceiling of her room. It can crawl on any surface except water and can drop vertically from any point. Maria puts each leg of her bed into a bowl of water to stop the tick from crawling up the legs. She then constructs another structure and happily sleeps through the night. What is it? The only constraint on the structure is that Maria and the tick remain in the same connected component of the air space.

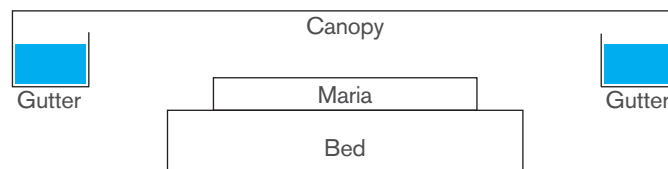
The consensus seems to be for an inward-facing gutter, as described by Ned Staples, my first department chair, who writes:

“Maria could have installed a large canopy with a short side curtain over her bed. Both are rigid. A gutter filled with water is attached on the lower inside edge of the curtain extending around the bed but beyond it.

“The tick will always be able to reach the top of the canopy, then the outside of the curtain, then the outside of the gutter. It cannot reach the inside wall of the curtain, nor the underside of the canopy. Since the bed is only under the canopy but not the gutter, it cannot drop onto the bed.

“The canopy can be supported either from the ceiling, by poles from the floor to the gutter, or by poles from its underside to the bed. The tick can reach the support in the first two cases but not the last.”

Below is a cross-sectional diagram. The water bowls for the bed legs are not included.



**BETTER LATE THAN NEVER**

**2011 S/O 1.** Timothy Chow notes that Sam Loyd’s name has only one  $L$ .

**OTHER RESPONDERS**

Responses have also been received from M. Branicky, A. Cann, F. Cann, J. Cornell, E. DeBonte, D. Erilas, H. Fowler, T. Hafer, J. Iler, M. Johnson, J. Korba, P. Kramer, B. Layton, J. Licini, N. Markovitz, D. McIlroy, T. Mita, J. Mohr, A. Ornstein, S. Poizeau, R. Ragni, P. Schottler, T. Sico, E. Signorelli, J. Sinnett, S. and H. Stern, M. Strauss, P. Sugar, R. Wake, and J. Yuan.

**PROPOSER’S SOLUTION TO SPEED PROBLEM**

$7,776 (n^{n-1})$

67 (the second-order difference between the numbers is 4) ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to [gottlieb@nyu.edu](mailto:gottlieb@nyu.edu). For other solutions and back issues, visit the Puzzle Corner website at [cs.nyu.edu/~gottlieb/tr](http://cs.nyu.edu/~gottlieb/tr).