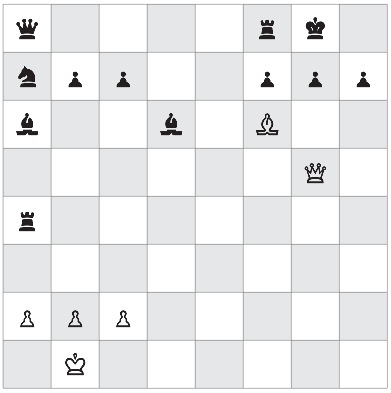
Time marches on, with some big numbers having just past by and others rapidly approaching. I have been at NYU for over 30 years and (when you read this) married for 40. Coming up in June is the 45th reunion of my MIT undergraduate class, which I hope to attend since I so very much enjoyed the 40th.

Finally, in 2015, I will turn 70 and Puzzle Corner will turn 50. The first is too frightening to contemplate; the second impossible to believe!

PROBLEMS

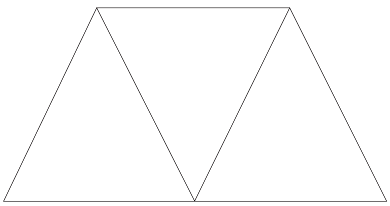
M/A 1. Jorgen Harmse offers what he calls "an anti-checkmate problem". Black is to play (good thing) and win.



M/A 2. Consider the trapezoid below constructed from three equilateral triangles.

We want to divide the trapezoid into 3 similar pieces (i.e., pieces of the same shape). Each piece must be connected and have a finite number of sides.

For part 1, all three must be of the same size; for part 2, two are the same size and the third different; and for part 3, all three have different sizes.



M/A 3. The late Bob High had sent us several cryptarithmetic puzzles. This month we present three. The first is his biblical offering.

ADAM * EVE = GARDEN

The second is a tribute to Nob Yoshigahara.

NOB + 1 = $\sqrt{\text{PUZZLE} + 1}$

The third, a tribute to him.

HIGH x HIGH + HIGH + HIGH + HIGH + HIGH = PUZZLES

SPEED DEPARTMENT

Sorab Vatcha has an equilateral triangle and wonders what is the ratio of the radii of the circumscribed and inscribed circles.

SOLUTIONS

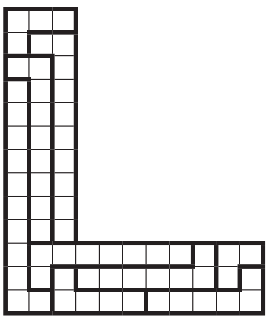
N/D 1. Surprisingly, North-South can make 7NT with only 1 high card point, but with two cooperative opponents. Rick Amerson sent us the following table, where a * indicates the winning card.

	N	E	S	W
1	9C	10C	JC*	8C
2	6S	AS	7C*	6C
3	2D	QS	5C*	4C
4	10S*	AH	5S	9S
5	8S*	KH	4S	7S
6	9H	7H	10H*	8H
7	3D	5H	6H*	KS
8	4D	3H	4H*	JS
9	5D	QH	3S*	AC
10	6D	JH	2S*	KC
11	7D	AD	2H*	QC
12	8D	QD	3C*	KD
13	9D	JD	2C*	10D

Amerson writes that first South runs three clubs while East kindly discards high spades and North wisely discards a spade to avoid later overtaking South. North then wins two spades. Next South takes three hearts while West discards spades. South then takes two spades while East discards hearts and West discards clubs. Finally West discards a club on South's 2H making South's last two clubs good, for 7NT.

N/D 2. Several readers commented that they very much enjoyed working on this puzzle. Jeffrey Schenkel, a self-proclaimed "old-fashioned guy", solved the problem by cutting out the pieces and physically mov- ing them around, starting with the largest piece. Since the criterion he gives for "old-fashioned" is being in the class of 1974, I guess my kids are correct in classifying me as antediluvian since I graduated even earlier.

Glenn Iba showed that the solution, shown below, is unique.



N/D 3. I received quite a number of fine solutions to this problem. Thank you all. In addition to supplying an analytic solution, Richard Hess notes that "Intuitively it can't fill up at time $T \neq \infty$ because, if we run the clock backwards from $t > T$, then there is nothing special about T that causes [the bowl] to start emptying at that point."

Burgess Rhodes generalized the problem to any shaped bowl. His complete solution is available on the puzzle corner web site, cs.nyu.edu/~gottlieb/tr.

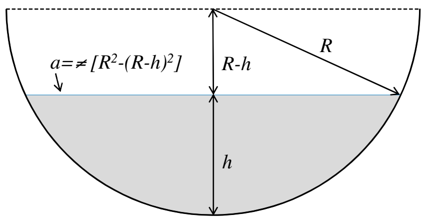
The following solution is from Apo Sezginer.

A hemispherical bowl of radius R is being filled with a liquid at a rate of r m^3/sec. The liquid evaporates at the rate ae m^3/sec where a is the surface area and e is the evaporation coefficient having units of m/sec. The evaporation and fill-rates are matched when the bowl is full: $r = e\pi R^2$

Let $h(t)$ be the depth of the liquid at the axis of the bowl, at time t. The bowl is empty at $t = 0$ when it starts being filled. The surface area is:

$a(h) = \pi\,(R^2 - (R-h)^2)$

as illustrated below.



The rate of change of volume equals the net of the fill and evaporation rates:

$$r - ae = e\pi\,(R-h)^2 = \frac{d}{dt}\int_0^{h(t)} a(h')\,dh' = a(h)h(t)$$

We obtain the differential equation:

$$h(t) = e\,\frac{(R-h)^2}{R^2-(R-h)^2}$$

Lets change the variable h to the dimensionless $u(t) = (R - h(t))/R$. The initial condition is: $u(0) = 1$. The differential equation becomes:

$$u(t) = \frac{e}{R}\,\frac{u^2}{u^2-1}$$

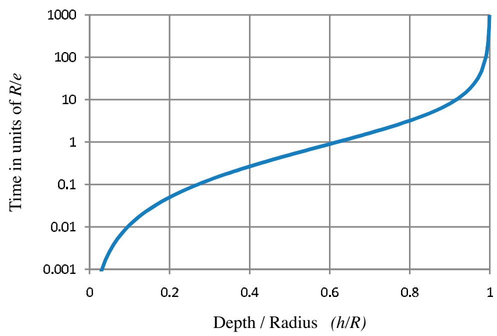
This is readily integrated to obtain:

$$t + c = \frac{R}{e}\left(u + \frac{1}{u}\right)$$

The integration constant c is determined by the initial condition $u(0) = 1$:

$$t = \frac{R}{e}\left(u + \frac{1}{u} - 2\right) = \frac{R}{e}(u-1)^2/u$$

As the bowl approaches being full, $h \to R$ and $u \to 0$. It is now clear that the bowl does not fill in finite time. Time versus depth is shown below in both equation and graphical form:



$$t = \frac{h^2}{(R-h)e}$$

$$h(t) = \frac{2R}{\sqrt{1+\frac{4R}{et}}+1}$$

BETTER LATE THAN NEVER

1982 M/A 4. Fred Tydeman believes that this problem is the same as problem #90 in *The Canterbury Puzzles* by H.E. Dudeney.

2011 J/A 2. Emil Friedman notes that this problem can be derived from example 51 on pages 39-41 of *A First Course in Probability, 2e* by Sheldon Ross.

OTHER RESPONDERS

Responses have also been received from R. Ackerberg, F. Albisu, P. Anderson, J. Feil, R. Freedman, R. Giovanniello, D. Griffel, J. Harmse, H. Hilhorst, P. Hochfeld, B. Houston, G. Iba, P. Kramer, A. Krishnan, G. Lesieutre, Z. Mester, F. Model, M. Nowitzky, T. Oktay, G. Perry, J. Prussing, E. Sard, J. Singer, E. Staples, H. Thiriez, D. Wachsman, S. Yang, and J. Yuan.

PROPOSER'S SOLUTION TO SPEED PROBLEM

2. Draw the picture and find the 30-60-90 triangle. These have side lengths 1, 2, $\sqrt{3}$. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.