It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

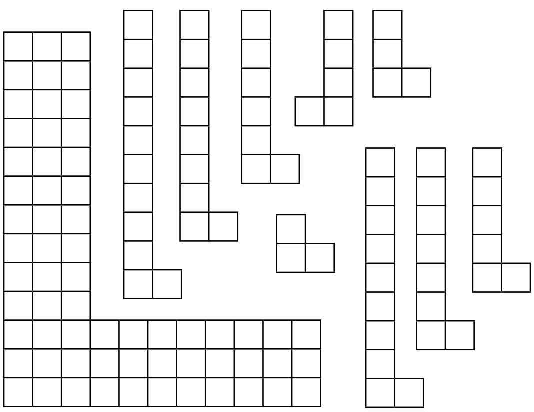
As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent via email, since these produce fewer typesetting errors.

PROBLEMS

N/D 1. Larry Kells finds that he does better as declarer when the opponents are on his side. He wonders what is the highest contract South can make as declarer assuming the defenders help as much as they legally can.

West	North	East
	♠ 10 8 6	
	♥ 9	
	♦ 9 8 7 6 5 4 3 2	
	♣ 9	
♠ K J 9 7		♠ A Q
♥ 8		♥ A K Q J 7 5 3
♦ K 10		♦ A Q J
♣ A K Q 8 6 4	**South**	♣ 10
	♠ 5 4 3 2	
	♥ 10 6 4 2	
	♦ —	
	♣ J 7 5 3 2	

N/D 2. Nob Yoshigahara wants you to put all the small L-shapes inside the large one. You may rotate a shape, but may not turn it over.



N/D 3. Ermanno Signorelli has an empty bowl with a hemispherical concavity. The plane of its lip is horizontal and the environment (temperature, pressure, humidity, etc) is constant. A liquid enters the bowl at a rate of r m^3/sec (cubic meters per second) and evaporates at a rate of ea m^3/sec, where a is the current surface area in meters squared (assume no meniscus). We can adjust r, but e is fixed.

Supposed r is chosen so that, when the bowl is full, the evaporation just matches the fill rate and hence the bowl remains full. What is the internal diameter of the bowl?

Using this same value of r, how long will it take to fill an initially empty bowl?

SPEED DEPARTMENT

Oren Helbok wants to know the one number that has all the letters in its English-language name in alphabetical order (non-consecutive, of course).

SOLUTIONS

J/A 1. Robert Wake and John Chandler submitted essentially the solution. These words are from Wake.

To avoid losing a trump trick against best defense, your side needs either at least 5 cards headed by at least AKJ, at least 6 cards headed by at least A Q 10 8, or at least 11 cards headed by at least AJ 5 points. With 4 suits and only 26 cards, that means 24 points is the best you can do. The minimum that makes a grand slam in all four suits with best defense is if each hand has A Q 10 8 in two suits and 432/32 in the other two, with all four KJ9 onside and all suits breaking evenly. That will provide abundant tricks and entries at both suits and notrump.

West	North	East
	♠ A Q 10 8	
	♥ A Q 10 8	
	♦ x x	
	♣ x x x	
♠ K J x		♠ x x x x
♥ K J x		♥ x x x
♦ x x x x		♦ K J x
♣ x x x	**South**	♣ K J x
	♠ x x	
	♥ x x x	
	♦ A Q 10 8	
	♣ A Q 10 8	

J/A 2. The following solution is from Donald Aucamp. An alternate solution from Apo Sezginer is on the Puzzle Corner website.

The probability, $P(n)$, no one gets their correct hat when n are returned at random is the sum of the first n+1 terms in the Maclaurin expansion of exp(x) at x = -1, as follows:

$$(1) \qquad P(n) = \sum_{0}^{n} (-1)\, k \,/\, k!$$

This gives P(1)=0, P(2)=1/2, P(3)=1/3, and P(4)=3/8, which are easy to check by listing all the permutations. Also, as P(5)=.366667 and P(6)=.368056, it is seen that P(n) very quickly approaches the limiting value of exp(-1) ≈ .367879.

By way of proof, a random selection of n hats can be viewed as one of $n!$ equally likely permutations of (1,2, ..., n). Let the vector, $V_n(x_1, \ldots, x_n)$ represent such a permutation. It is feasible (i.e., no one gets their right hat) if $x_i \neq i$ for all i. Accordingly,

$$(2) \qquad P(n) = M(n) \,/\, n!$$

where M(n) is the number of feasible V_n. It is shown below that M(n) satisfies

$$(3) \qquad M(n+1) = n\, M(n) + n\, M(n-1)$$

Since M(1)=0 and M(2)=1 are trivially found, then all M(n) can be determined by iteration. The solution below, which can be checked by inserting it into (3), is

$$(4) \qquad M(n) = n! \sum_{0}^{n} (-1)^k \,/\, k!$$

Equations (4) and (2) imply (1).

To prove (3) define the operator E_i, which transforms V_n into V_{n+1} by replacing x_i with n+1 and tacking on x_i at the end. For example, suppose n=3 and V_3=(2,3,1). Then E_2V_3=(2, 4, 1, 3), where the second element gets replaced by n+1 (i.e., 4), and the exiting 3 goes into the last position. Note that every V_{n+1} can be derived from some E_iV_n as follows: In V_{n+1} assume n+1 is element i. Interchange this with the last element, x_{n+1}, and let V_n be the vector containing the first n elements. Then $E_iV_n = V_{n+1}$. A little thought shows that a feasible V_{n+1} can arise from some E_iV_n operation in only one of two ways:

(a) Start with one of the M(n) feasible V_n, choose any i ($1 \leq i \leq n$) and let $V_{n+1} = E_iV_{n+1}$. Then V_{n+1} will be feasible, and there are nM(n) ways of doing this. This is the first term in (3).

(b) Start with any singly infeasible V_n (a permutation with exactly one offending element). Say this is the i^th element, so that $x_i = i$, and set $V_{n+1} = E_iV_n$. Then V_{n+1} will be feasible. As there are n possible offending elements, and as there are M(n-1) possible V_n vectors yielding this condition (which are found by looking at all the feasible permutations after that element is erased), then there are nM(n-1) vectors with this property. This is the second term in (3). Thus, (3) is proved.

For example, suppose n=3 and note M(2)=1 and M(3)=2. Then, for n+1=4, Equation (3) yields M(4)=3M(3)+3M(2)=9, which can readily be checked by listing all the 4!=24 permutations and picking off the nine feasible solutions. These nine solutions can be found from the set of possible V_3 vectors as follows: The two feasible solutions are (2,3,1) and (3,1,2). The first of these leads to n=3 solutions, which are E_1(2,3,1)=(4,3,1,2), E_2(2,3,1)=(2,4,1,3), and E_3(2,3,1)=(2,3,4,1). The second yields E_1(3,1,2)=(4,1,2,3), E_2(3,1,2)=(3,4,2,1), and E_3(3,1,2)=(3,1,4,2). Moreover, there are three singly infeasible V_3 solutions, which are (1,3,2), (2,1,3), and (3,2,1). These lead to three feasible V_4 solutions: E_1(1,3,2)=(4,3,2,1), E_3(2,1,3)=(2,1,4,3), and E_2(3,2,1)=(3,4,1,2). In conclusion, the two feasible V_3 vectors and three singly infeasible V_3 vectors lead to the nine V_4 feasible vectors, and Equation (3) is confirmed.

J/A 3. David Zagorski was especially happy to solve this problem since he is a friend of Dick Hess and they meet each year at the US Tennis Open. Zagorski writes.

A's response means that B and C are not both wearing 11's. B's response excludes the possibility that A and C are both wearing 11's. In addition, B knows that, if C was wearing 11, then A's response would require B to be wearing 7. B's response excludes this possibility also. Therefore C must be wearing 7.

BETTER LATE THAN NEVER

1996 M/J 1 Benard Lemaire has sent us a tech report showing solutions to the non-dominating N Queens problem, which now appears on the puzzle corner web page (cs.nyu.edu/˜gottlieb/tr).

2011 M/A 1 Randall Pratt notes that his local newspaper's sudoku puzzle yielded essentially the published solution and both arrived at his home the same day.

M/A 3 Robert Ackerberg noticed that the origin (0,0) should be shifted to the left so that it is under the center of the far left semi-circle. Burgus Rhodes's generalization is on the website.

J/A SD Robert Mandl, Naomi Markovitz, Burgess Rhodes and Harvey Lynch objected to the published solution. Rhodes's detailed solution appears on the website.

OTHER RESPONDERS

Responses have also been received from S. Berger, C. Charoen-Rajapark, T. Chow, E. Collins, G. Coram, J. Desmond, R. Giovanniello, J. Hardis, B. Haris, A. Hirshberg, H. Hodara, S. Korb, J. Korba, E. Kutin, P. Lemieux, W. Lemnios, M. Lenot, F. Model, S. Nason, A. Ornstein, P. Paternoster, M. Piazza, J. Prussing, Z. Rifkin, B. Rorschach, L. Schaider, M. Seidel, I. Shalom, S. Silberberg, C. Travares, and K. Zeger.

PROPOSER'S SOLUTION TO SPEED PROBLEM

40. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.