Some sad news to report. I heard from Michele Smith that her husband, longtime contributor Joel Karnofsky, died in late March. I remember well several of his thoughtful contributions and wish his family peace in this time of grief.

It is mid-April as I write, and spring has finally come to the northeastern United States. The last slow melted weeks ago; temperatures in the 60s are common and 70s have been recorded.

We can only wish that correspondingly sunny days shine on the trouble spots in the Middle East and Japan.

PROBLEMS

J/A 1. Larry Kells wonders: what is the smallest number of combined high-card points that will allow North-South to make a grand slam in any suit or no-trump from either side of the table against best defense?

J/A 2. Harold Ingraham attends classier social events than I do. At his, all attendees wear hats, and a hat-check person is there to check them. But once, they became hopelessly scrambled and everyone received a random hat from the pile. Assuming all permutations are equally likely, what is the probability that no one got back the right hat?

J/A 3. On a related theme, we offer another of Dick Hess's "logical hat" problems, in which logicians see the numbers on every other logician's hat but not on their own. Each logician reasons correctly and knows that the others do as well.

In this comparatively easy example there are five slips of paper; 7 is written on three of them and 11 on the other two. One goes on logician A's hat, a second on B's, a third on C's, and the other two are hidden. First A says, "I don't know my number." Then B says, "I don't know my number."

What is C's number?

SPEED DEPARTMENT

George Bloom wishes to drill a hole clear through the center of a solid sphere. His flat-bottom drill is exactly six inches long, and he uses all of it in the drilling operation. How much material is left?

SOLUTIONS

M/A 1. Our first problem is from Lorraine Mullin, who writes, "Consider the solution of any ordinary sudoku puzzle. What is the largest number of times that any fixed integer can appear on either of the two major diagonals? Show that this number can actually occur."

Ken Zeger notes that five is an upper bound, since the two main diagonals pass through only five 3 × 3 subsquares, and a given number appears exactly once in each of the subsquares.

His young son Kai produced the following sudoku solution with five highlighted 1s on the main diagonals, achieving the above bound and thus showing that five is indeed the answer.

3	2	5	9	6	1	4	8	7
6	1	8	2	7	4	9	3	5
9	7	4	8	3	5	1	6	2
4	8	3	7	2	9	5	1	6
2	9	7	5	1	6	8	4	3
1	5	6	4	8	3	7	2	9
7	4	1	3	5	2	6	9	8
5	6	2	1	9	8	3	7	4
8	3	9	6	4	7	2	5	1

M/A 2. A "politically correct" problem from my NYU colleague Joel Spenser.

On an infinite chessboard an Obamaknight can move six spaces in any direction, then turn left and move one space. For example, from (5,−3) he can move N-W to (4,3) followed by two moves E-N ending at (16,5). Place a finite number of Obamaknights on the board so that (allowing an arbitrary number of moves):

(i) no one of them can reach any other of them.

(ii) any position on the board can be reached by one of them.

Extension: A modern Obamaknight moves six spaces in any direction, then turns left or right and moves one space. Place a finite number of modern Obamaknights with the above properties.

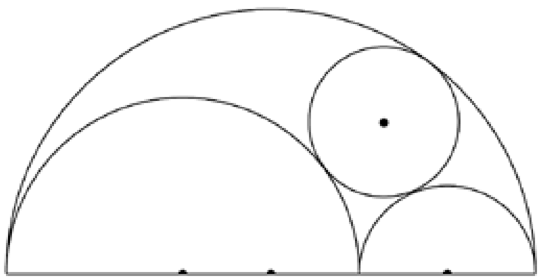
The proposer shows that placing 37 Obamaknights in a line solves the original version. Mark Perkins and Aaron Ucko gave "square plus one" solutions. Perkins's solution follows.

"Thirty-seven Obamaknights are required to fulfill condition (ii). To fulfill condition (i), they can be arranged in a 6 × 6 square with an additional Obamaknight below the leftmost column of the square. If one considers this block moving as a unit (according to the 6 + 1 method), it can be seen that this 37-square figure will tile the plane.

"For the extension, it turns out that only one modern Obamaknight is required to fulfill condition (ii) and, clearly, the one modern Obamaknight will fulfill condition (i) (since one piece cannot attack itself). To see that one is sufficient to meet condition (ii), first note that by two (back-and-forth) moves—e.g., (0,0) -> (1,6) -> (2,0)—the piece can be moved two squares to the right (or left or up or down). Thus, if we start at (0,0) we will easily cover all squares in which both coördinates are even. However, by focusing on our position after the first move—to (1,6)—we can also then cover all positions in which the first coördinate is odd but the second is even (by using our dual-move two-square hop). Then, by considering the first move to the right instead of up—to (6,1)—we can similarly cover positions for which the first coördinate is odd and the second coördinate is even. Finally, by two moves at right angles to each other—e.g., (0,0) -> (1,6) -> (7,7)—we can reach a position from which we can then reach all (odd, odd) positions.

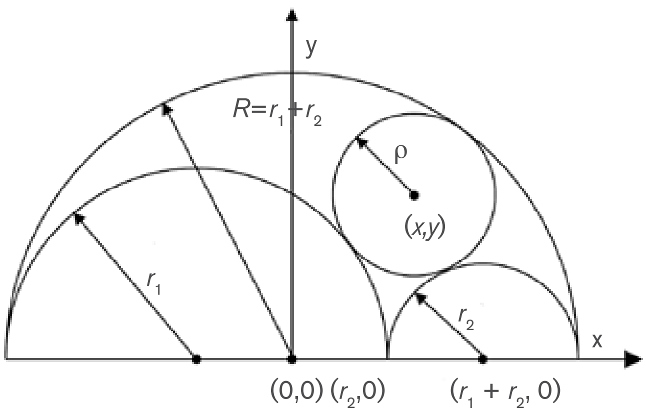
"I guess the moral of the story is that just one bipartisan piece is able to accomplish the same mission as 37 partisan pieces!"

M/A 3. The top figure below, from John Craig, contains three semicircles and one circle. The semicircles all have horizontal diameters, and their centers are shown. Their radii are R, r_1, and r_2 with $r_1 + r_2 = R$. The circle is constructed tangent to the three semicircles.



Find ρ, the radius of the circle, and show that the distance from its center to the base line is 2ρ.

I received several fine solutions to this problem. Peter Lunquist's, based on the law of cosines, is on the Puzzle Corner website; William Lemnios's follows.



Place the centers of the three semicircles on the x-axis of a Cartesian coördinate system whose origin is at the center of the left semicircle. The figure above shows the radii and the coördinates of the centers of the three semicircles and the tangent circle.

From this figure the following relationships can be established:

$$x^2 + y^2 = (r_1 + \rho)^2 \tag{1}$$

$$(r_1 + r_2 - x)^2 + y^2 = (r_2 + \rho)^2 \tag{2}$$

$$(x - r_2)^2 + y^2 = (r_1 + r_2 - \rho)^2 \tag{3}$$

Eliminate y^2 from (1) and (2) and eliminate y^2 from (1) and (3) to obtain

$$(r_2 - r_1)\rho + (r_1 + r_2)x = r_1^2 + r_1 r_2 \tag{4}$$

$$(2r_1 + r_2)\rho - r_2 x = r_1 r_2 \tag{5}$$

The solution to these equations is

$$x = \frac{r_1^2(R + r_2)}{R^2 - r_1 r_2} \tag{6}$$

$$\rho = \frac{r_1 r_2 R}{R^2 - r_1 r_2} \tag{7}$$

the radius of the circle.

$$R = r_1 + r_2 \tag{8}$$

By substituting (6) and (7) into (1) and grinding through a pile of symbols, we can solve for y

$$y = \frac{2 r_1 r_2 R}{R^2 - r_1 r_2} = 2\rho \tag{9}$$

the height of the circle's center above the x-axis.

BETTER LATE THAN NEVER

M/A SD Francisco Albisu, Eva Jansson, Ray Schnitzler, and Roy Swart note that the quotient is close to 5π, not π/2. A few other readers sent in solutions, apparently not realizing that speed problems are answered at the end of the column in which the problem appears.

OTHER RESPONDERS

Responses have also been received from G. Bergman, G. Blum, B. Brademeyer, L. Cutrona, D. de Champeaux, C. Daily, J. Feil, R. Giovanniello, T. Harriman, T. Harriman, O. Helbok, H. Hodara, W. Jasper , D. Katz, J. Kramer, Z. Mester, T. Mita, F. Pollitz, E. Sard, E. Sard, T. Schonbek, S. Shapiro, T. Sim, C. Swift, T. Tu, and T. Wilson.

PROPOSER'S SOLUTION TO SPEED PROBLEM

36π. Since the diameter of the drill isn't given, the answer must be independent of it. So consider an "infinitely thin" drill *[we don't say "limit" in a speed problem—Ed.]*. All the sphere is left, and its volume is $(4/3)\pi 3^3$.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.