Last week, on 7 January, my beautiful wife Alice and I celebrated our 39th wedding anniversary. Nature cooperated by delivering our second snowstorm this winter and turning our property and neighborhood into a winter wonderland. Fortunately, we did not have to drive anywhere for a few days and could admire the picture postcard scenes from our home and from daily walks.

Whenever, we get significant snow (about 3.5' for those two plus last night's storm), I remember the "real snow" we had one year around 1970 in Waltham/Boston: 30" one week, 24" the next, and some more the week later. I must add, however, that my younger son Michael and his fiance'e Maureen live in Syracuse NY and, to put it mildly, are *not* impressed by any of these storms.

PROBLEMS

M/A 1. Our first problem is from Lorraine Mullin who writes. Consider the solution of any ordinary Sudoku puzzle. What is the largest number of times that any fixed integer can appear on either of the two major diagonals and show that this number can actually occur?

M/A 2. A "politically correct" problem from my NYU colleague Joel Spenser.

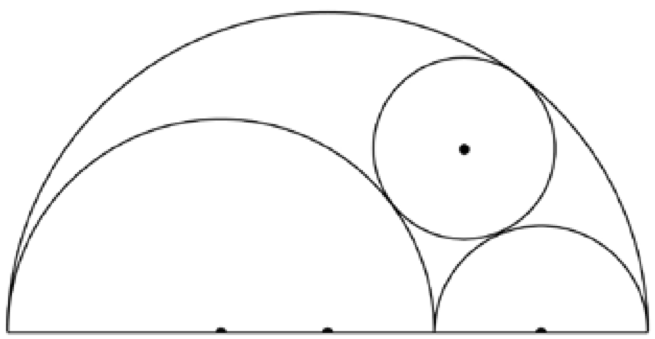
On an infinite chessboard an Obamaknight can move six spaces in any direction followed by turning left and moving one space. For example, from (5,-3) he can move N-W to (4,3) followed by two moves E-N ending at (16,5). Place a finite number of Obamaknights on the board so that (allowing an arbitrary number of moves)

(i) no one of them can reach any other of them.

(ii) any position on the board can be reached by one of them.

[Extension:] A modern Obamaknight moves six spaces in any direction followed by turning left or right and then moving one space. Place a finite number of modern Obamaknights with the above properties.

M/A 3. The figure below, from John Craig, contains 3 semicircles and one circle. The semicircles all have horizontal diameters and their centers are shown. Their radii are R, r_1, and r_2 with $r_1 + r_2 = R$. The circle is constructed tangent to the three semicircles. Find ρ, the radius of the circle, and show that the distance from its center to the base line is 2ρ.



SPEED DEPARTMENT

Sid Shapiro wants to know what is special about 8,549,176,320?

SOLUTIONS

N/D 1. Larry Kells wants to know what is the fewest high card points that East-West can have (their hands are to be specified) to guarantee that North-South cannot make a game in any denomination (including no trump), regardless of the distribution of the N-S cards?

Kells himself sent us the hand below with the comment that there are enough trump tricks to sink any suit game and five space tricks against 3NT.

East	West
♠ A K Q J 10	♠ 2
♥ 10 9 8 7 6 5 4 3	♥ —
♦ —	♦ J 10 9 8 7 6
♣ —	♣ J 10 9 8 7 6

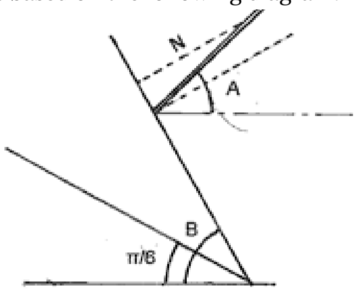
Chet Sandberg notes that, if we are allowed the specify all four hands, the deal below, with only 7 points for the defenders, yields no game. Sandberg attributes the hand to "Bridge in the the Menagerie" by Victor Mollo, a book he recommends.

	North	
	♠ K Q	
	♥ K J	
	♦ 10 6 5 4 2	
East	♣ 9 7 5 4	West
♠ 10 9 7 6 5		♠ 8 4 3 2
♥ 5 4 3 2		♥ 10 9 8 7 6
♦ K J 3		♦ —
♣ 6	South	♣ Q J 10 8
	♠ A J	
	♥ A Q	
	♦ A Q 9 8 7	
	♣ A K 3 2	

N/D 2. Bill Deane's son Mike drives an old Jeep that has sustained at least 4 cracked windshields. Mike contends that it is due to the steepness of the Jeep windshield compared to most other cars.

Stones impact at various angles A from the horizontal. For which values of A is the normal component of the force on a Jeep windshield (which we assume is sloped 60 degrees from the horizontal) greater than on a more normal windshield (sloped 30 degrees from the horizontal).

F. Albisu sent us both analytic and graphical solutions. His analytic solution is based on the following diagram.



If F is the magnitude (force, momentum, or whatever) quantifying the impact strength, its component normal to the windshield with angle B (see the left figure below) is

$N = F \sin(\pi - (B + A)) = \sin(B + A)$

which, for the windshield at 30° (the standard), becomes

$F \sin(\pi/6 + A)$.

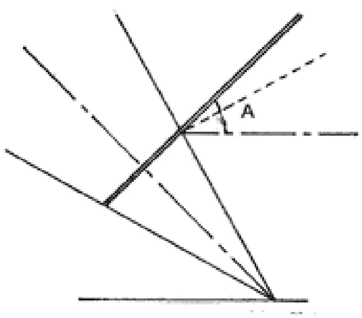
The condition of more damage to the windshield (i.e., larger normal component) at angle B than at 30° becomes

$\sin(B + A) > \sin(\pi/6 + A)$.

Simple trigonometry yields $\tan A < (\sin B - 1/2)/(\sqrt{3}/2 - \cos B)$.

For Mike's Jeep, with $B = 60°$, this inequality leads to $A < 45°$ with respect to the horizontal, which includes of course the directions $A < 0$ below it.

Albisu's graphical solution is based on the figure below. He notes that the angle A satisfying the stated condition is the direction perpendicular to the bisector of the angle formed by the two windshield positions being considered.

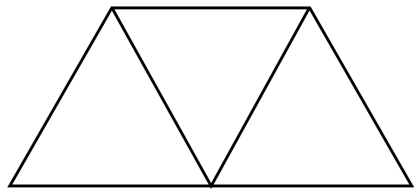


For this value of A (which for this problem can be deduced directly very easily given the 30° and 60° positions (and which has been also calculated above as 45°), the angle of incidence, and therefore the normal component, is seen to be the same for the two windshields. Turing the stone direction clockwise increases that component on the steeper windshield while decreasing it on the other one. The contrary happens with a conter-clockwise turning.

Ted Mita believes we should also consider stones ricocheting of the hood.

N/D 3. Richard Hess sent us several tiling problems from Bob Wainwright. Here are the first two. If they are popular, other (I believe more difficult) versions will appear in later issues.

You are to dissect the equilateral trapezoid below composed of three equilateral triangles into two similar parts (i.e., parts of the same shape). Each part must be connected and have a finite number of sides. For the first problem, the parts must of the same size, and for the second problem the parts must be of different sizes.



Avi Ornstein writes that to be similar and the same size, the parts must be congruent, which can be done by dissecting the figure vertically in the middle.

To have the pieces similar but of different sizes, use a horizontal dissection. The top of the total trapezoid is one unit in length, the bottom two units. Since the resulting two trapezoids are to be similar, they must have the same ratio of top to bottom lengths. If the dividing line is x units long, the ratios are $1/x$ and $x/2$. Setting these equal gives $x = \sqrt{2}$, which defines where the original trapezoid must be dissected.

BETTER LATE THAN NEVER

2010 J/A 2. Joel Karnofsky notes that his solution, which appears in on the puzzle corner web site cs.nyu.edu/˜gottlieb/tr has more distinct solutions that one published in the column itself.

J/A 3. Jake Yara and Steven Pudar note that the correct answer for 125 is $25^{9/6}$.

OTHER RESPONDERS

Responses have also been received from P. Cassady, R. Currier, J. Feil, R. Giovanniello, Y. Hinuma, P. Kumbhat, G. Lobdell, M. Perkins, K. Rosato, E. Sard, A. Shuchat, E. Signorelli, C. Tavares, and G. Wood.

PROPOSER'S SOLUTION TO SPEED PROBLEM

It is the digits arranged in English alphabetical order. Shapiro notes that, when this 10 digit value is divided by the Spanish equivalent, the quotient is quite close to $\pi/2$. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.