This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 1) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2010 yearly problem is in the "Solutions" section.

I have just returned from the wedding of our older son, David, and his lovely bride, Marissa Williams. This issue is dedicated to their long and happy marriage.

PROBLEMS

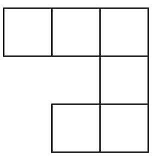
Y2011. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 1 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 1 are preferred. Parentheses may be used and do not count as operators. A leading minus sign does count as an operator.

J/F 1. A quite different bridge problem from Larry Kells: one that involves scoring, not playing. He writes,

"In tournament team play, deals are played at two tables and the two scores are compared. If the scores are different, the difference of the two North-South scores (treated as a positive number) is called the swing. For the purpose of calculating the difference, a positive East-West score is treated as a negative North-South score. Current tournament scoring of part scores, games, over- and undertricks, etc., is used. Honors are not counted.

"What is the smallest positive multiple of 10 points that cannot be the exact swing on a single deal? How about 50 or 100 points? You are not being asked to give hands and reasonable bidding sequences and reasonable play."

J/F 2. This problem is another in the series sent by Richard Hess and Robert Wainwright. Find a way to connect tiles so that five of them cover at least 92 percent of the hexomino below. The tiles must be identical in size and shape and may be turned over so that some are mirror images of the others. They must not overlap each other or the border of the hexomino.



SPEED DEPARTMENT

Alan Faller was born on the day Herbert Hoover was sworn in as president of the United States. Express his age on the next Presidents Day in powers of 10.

SOLUTIONS

Y2010. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 0 exactly once each and the operators +, −, × (multiplication), / (division), and exponentiation? We seek solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 0 are preferred. Parentheses may be used for grouping and do not count as operators. A leading minus sign does count as an operator. Zero to the zero power is not permitted.

The following solution is from Ermanno Signorelli.

1	201^0	12	2+0+10
2	20/10	19	20-1+0
3	$2+10^0$	20	20+1*0
5	10/2+0	21	20+1+0
8	10-2-0	30	20+10
9	$10 - 2^0$	50	100/2
10	20-10	98	100-2
11	$2^0 + 10$	100	$10^2 + 0$

Joel Karnofsky has analyzed the century and found that the best year is 2069 (or 2096), with 70 solutions. The worst was 2000, with three. The very best four-digit years are 1389 (I remember it well), 2347, and 2379 (or permutations), with 98 solutions.

S/O 1. Larry Kells wants to know the fewest high points a single bridge player can have and still be sure of beating 3 no-trump. What about 1 no-trump?

Apparently this problem is either not too hard (if you can assume that as opponent you are on lead) or rather difficult (if you cannot).

In the former case, we have the following, from Doug Foxvog.

"I'm not sure why this is not a speed problem. The answer seems simple: seven, seven. To be sure of beating 3 no-trump, an opponent needs to know he will take at least five tricks. If the opponent does not take these at the start, then he must be able to block the declarer from running a single suit or playing nine high cards off the top. To perform such a block, the opponent would need at minimum all queens and jacks or 12 points. To block 1 no-trump would take more points. However, with AK10xxxxxxxx x x void or AQJ10xxxxxxxx x void void (both seven high-card points), the opponent can run the first 11 or 12 tricks, defeating any NT contract."

Without assuming the lead, the only response was from the proposer, who writes,

"The following 18-point hand can hold the opponents to five tricks at no-trump: A A A10987 KQJ1098. If partner is on lead and has no clubs, he should lead a diamond. You can win and set up the clubs, and diamonds are still stopped. If partner is void in both minors, he should lead his longer major. You win and set up the clubs, and declarer has at most four cashable tricks in the suit partner led (since partner had at least seven in that suit)."

S/O 2. As M/A 2 noted, 4,159 is the first four-digit prime to occur in the expansion of pi (it starts at the third digit) and 5,926,535,897 is the first 10-digit prime to occur (it starts at the fifth digit). Eric Nelson-Melby asks two questions. What is the largest prime starting at the first position of the expansion? Which $n \le 400$ requires going the furthest into the expansion to find an n-digit prime?

For the second question, Joel Karnofsky (and, I suppose, some computer) found that the first prime with 305 digits starts at the 4,056th digit, which is the furthest required. For part 1, he found primes as an initial sequence of pi of lengths 1, 2, 6, 38, and 16,208. He then led me to www.research.att.com/~njas/sequences/A060421, which includes the above plus the lengths 47,577 and 78,073.

Jorgen Harmse (see Berend and Harmse, "On Polynomial-Factorial Diophantine Equations," *Transactions of the American Mathematical Society* 38(4): 1741–1779) adds some theory to the discussion by writing, "Perhaps it is just as well that Eric Nelson-Melby did not ask for a proof of maximality: I conjecture that there are infinitely many primes starting at the first position (or any other position) in the expansion with any base of pi (or almost any other number). Moreover, I expect that the number of such primes up to n digits is about $\log_b(n)$, where b is the base of the expansion.

"Number theorists sometimes treat the primes as if they were randomly distributed. For example, if you divide almost any prime by 6 the remainder will be 1 or 5, and Dirichlet's theorem says that, asymptotically, each possible remainder occurs half the time. What follows is thus not a proof of my conjecture but an argument that most number theorists would find persuasive. (I assume that the digits of pi can also be treated as random. The digits of most numbers between 3 and 4 will satisfy any given test of randomness, and pi is defined by an integral rather than by any property of its digits.)

"By the prime number theorem, the 'probability' that an integer close to x is prime is asymptotically $1/ln(x)$. A k-digit number is between b^{k-1} and b^k, so (for large k) the probability that it is prime is between $1/(k\ln(b))$ and $1/((k-1)\ln(b))$. Thus the expected number of primes up to n digits starting at a given position in the expansion of pi (even if there is a string of zeros at that position) is

$$O(1) + 1/\ln(b) + 1/(2\ln(b)) + \ldots = O(1) + \ln(n)/\ln(b) = O(1) + log_b(n).$$

"In particular, we should expect infinitely many primes beginning at any given position."

S/O 3. Robert Ackerberg uses mirrors (but not smoke) when doing number theory. He notes that some "mirror numbers" have "mirror squares." For example, consider 12 and its "mirror" 21 and note that their squares ($12^2 = 144$ and $21^2 = 441$) are mirrors. This holds for 13 and its mirror 31 but not for 14 and 41. What three-digit numbers (e.g., 113 and 311) have this property? What about four?

Although this was not formally part of the problem, Richard Hess reports that the two-digit solutions are 11, 12, 13, and 22.

William Lennios sent us an analysis that can be found on the Puzzle Corner website, cs.nyu.edu/~gottlieb/tr. The three-digit and four-digit solutions are as follows (*nRev* is the reverse of n).

n	n^2	$nRev$	$nRev^2$
101	10,201	101	10,201
102	10,404	201	40,401
103	10,609	301	90,601
111	12,321	111	12,321
112	12,544	211	44,521
113	12,769	311	96,721
121	14,641	121	14,641
122	14,884	221	48,841
202	40,804	202	40,804
212	44,944	212	44,944
1,001	1,002,001	1,001	1,002,001
1,002	1,004,004	2,001	4,004,001
1,003	1,006,009	3,001	9,006,001
1,011	1,022,121	1,101	1,212,201
1,012	1,024,144	2,101	4,414,201
1,013	1,026,169	3,101	9,616,201
1,021	1,042,441	1,201	1,442,401
1,022	1,044,484	2,201	4,844,401
1,031	1,062,961	1,301	1,692,601
1,102	1,214,404	2,011	4,044,121
1,103	1,216,609	3,011	9,066,121
1,111	1,234,321	1,111	1,234,321
1,112	1,236,544	2,111	4,456,321
1,113	1,238,769	3,111	9,678,321
1,121	1,256,641	1,211	1,466,521
1,122	1,258,884	2,211	4,888,521
1,202	1,444,804	2,021	4,084,441
1,212	1,468,944	2,121	4,498,641
2,002	4,008,004	2,002	4,008,004
2,012	4,048,144	2,102	4,418,404
2,022	4,088,484	2,202	4,848,804

OTHER RESPONDERS

Responses have also been received from G. Chan, C. Dale, R. Ellis, J. Feil, A. Hirschberg, M. Kay, R. Marks, A. Ornstein, M. Perkins, J. Prussing, K. Rosato, P. Schottler, T. Sim, A. Taylor, and J. Worr.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Alan's birth date is March 4, 1929, and on Presidents Day he will still be 81 (decimal), which is $10^{110} + 10^{100} + 10^{0}$ (binary). Sorry for the U.S.-centric knowledge needed. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.