

Since this is the first issue of an academic year, let me review the ground rules. In each issue I present three regular problems, the first of which is normally related to bridge (or chess or some other game), and one “speed” problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e., four months) later, one solution is printed for each; I also list other readers who responded. For example, the current issue contains solutions to the problems posed in May/June.

The solutions to the problems in this issue will appear in the January/February column, which I will need to submit in mid-October. Please try to send your solutions early to ensure that they arrive before my deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the “Other Responders” section. Major corrections or additions to published solutions are sometimes printed in the “Better Late Than Never” section, as are solutions to previously unsolved problems.

For speed problems, the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution, it appears at the end of the column in which the problem is published. For example, the solution to this issue’s speed problem is below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that remained unsolved.

PROBLEMS

S/O 1. Larry Kells wants to know the fewest high points a bridge player can have and still be sure of beating 3 no-trump. What about 1 no-trump?

S/O 2. As noted in M/A 2, 4159 is the first four-digit prime to occur in the expansion of pi (it starts at the third digit of the expansion) and 5,926,535,897 is the first 10-digit prime to occur (it starts at the fifth digit). Eric Nelson-Melby asks two related questions. What is the largest prime you can find starting at the first position of the expansion (no proof of maximality expected)? Which $n \leq 400$ requires going the furthest into the expansion to find an n -digit prime?

S/O 3. Robert Ackenberg uses mirrors (but not smoke) when doing number theory. He notes that some “mirror numbers” have “mirror squares.” For example, consider 12 and its “mirror” 21 and note that their squares $12^2 = 144$ and $21^2 = 441$ are mirrors. This holds for 13 and its mirror 31 but does not hold for 14 and 41. What three-digit numbers (e.g., 113 and 311) have this property? What about four-digits?

SPEED DEPARTMENT

Mark Astolfi wonders, how someone born in this millennium can be older than someone born in the previous millennium?

SOLUTIONS

M/J 1. There are two rather different approaches. One is to duck the opening lead; the other is to win it. The variation is caused primarily by different opinions as to the likely distribution of the remaining cards. I present one solution from each camp. Representing the “duckers,” we have the following response from Len Schaidler:

“After seeing the opening lead and dummy, I know that West does not have the spade ace; if so, he would have led it, then switched to a diamond, and we would have four tricks. Based on the bidding, if South has the king of hearts, then declarer would be able to make the contract easily. So I assume the West and South distributions are

West
 ♠ J 10 8 6
 ♥ K J 9 4
 ♦ 3
 ♣ J 9 6 4

South
 ♠ A 9
 ♥ 10 7 2
 ♦ 10 9 5 4
 ♣ A K 8 7

“North-South has nine sure tricks—three spades, two hearts, one diamond, and three clubs—and needs one more to make the contract. I can take the opening lead, cash two more diamonds, and exit with a diamond. But that will cause my partner to make discards, in front of the dummy; some of these could be winners, and South can wisely choose discards from dummy based on what West discards. To avoid making West discard potential winners and since South has a diamond winner no matter what I do, I merely play low on the first trick. Since North-South has only nine sure tricks, I could take four diamond tricks if I ever get the lead. Since North wins the first trick with the king of diamonds, South must plan his method of attack. The only way to make the contract is to win three heart tricks. If South ever leads a heart, West’s KJ9 in front of dummy will only allow North-South to win two tricks with the ace and queen, even if South tries two finesses through West. His best approach is to cash good clubs and/or spades and then give West the lead with a club or spade. West will take two tricks with his black jacks but must lead a heart. As long as West either leads the king or jack, North-South will be limited to two heart tricks. South could even duck a heart lead by West, but it does not matter. The key things are that West must not lead the 9 or 4 of hearts (this would allow South’s 10 of hearts to win a trick) and that South’s 10 of hearts will be gone after the third round and West’s 9 of hearts will be high, no matter how the hearts were played. So West will win four tricks: one spade, one club, and two hearts. If South leads a diamond, I win four tricks.

“The key is that by ducking the opening lead, either West wins four trick or I win four tricks, and we defeat the contract; in either case, I or my partner are discarding winners on our partners’ winners!”

Dudley Church, representing the “winners,” writes:

“The basic problem is to find the probable South hand, based on the bidding. My solution is:

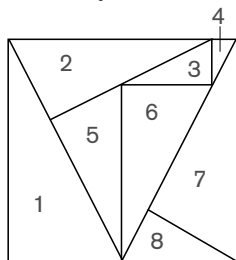
- ♠ A J
- ♥ K x
- ♦ 10 9 x x
- ♣ A J x x x

“South has a legitimate opening bid of one club. North bids his four-card heart suit, and East sticks in his spoiler of three diamonds. South has a minimum opening bid and has no support for Hearts until he gets more information, so he passes. West passes and North bids three spades. After East passes, South figures that North has four hearts and four spades, with five cards split between diamonds and clubs. Since East must have six or seven diamonds, North at the most would have two or three diamonds. If either North or East has one diamond face card, then the South diamond 10 will keep East from running his diamonds. Therefore, South bids three no-trump. When East bids four diamonds, South passes, because he has bid all he can with his hand. East’s bid of four diamonds is strictly defensive in order to keep the opponents from making three no-trump or driving them up to four no-trump, which North bids.

“Now when East sees the Dummy, and deduces what South is most likely to hold, his best chance is to take the ace, king, and queen of diamonds, and then lead the club 10, expecting that West’s club king will take the setting trick.”

M/J 2. The proposer sent us the diagram below with the assertion that 8 is the minimum number known, suggesting that this is still an open problem. I was intrigued by the diagram and surprised that all these triangles had side lengths $x-2x-\sqrt{5}x$. If my calculations are correct, then the values of x for the triangles as numbered, assuming that the square is 10×10 , are 5, $(9/5)\sqrt{5}$, 2, 1, $(8/5)\sqrt{5}$, 4, $2\sqrt{5}$, and $\sqrt{5}$.

Note that the hypotenuse of triangle 3, when extended, is perpendicular to the hypotenuse of triangle 1, since their slopes are negative reciprocals.



M/J 3. This was a very popular problem. I must admit to being quite surprised to find that the gnomes could do so well. Several readers had schemes whereby the gnomes encoded extra information in their responses (loudness, rapidity, inflection, etc.). However, no such ploys are necessary, as the following parity-based solution from Walid Nasrallah illustrates.

“I had fun with the gnome problem. I knew instantly that all the gnomes could be saved except for gnome 1, the one in the rear, who is the first one to answer. This gnome, who has no help from

the others, must face a 50-50 chance of dying. The strategy below works for any number of gnomes, not just eight, but if the number of gnomes is not known in advance, then the witch can defeat the plan by impersonating just one extra nonaltruistic gnome.

“The simplest statement of the solution strategy that I could think of was: ‘Every gnome counts the number of black hats in front of him, plus the number of times he has heard “black” before, and says “black” if the sum is odd and “white” if it is even.’

“To show the strategy works, we use induction.

“Base case: observe that gnome 2 has either a white hat, meaning that all the black hats he sees were also seen by gnome 1, or a black hat, meaning that gnome 1 saw one more black hat than he (gnome 2) sees. In both cases, if the two counts match, then their sum must be even and, if the two counts differ, then they differ by one and their sum must be odd. So saying ‘black’ for an odd sum and ‘white’ for an even sum will save gnome 2’s life.

“Induction step: If all the gnomes before gnome n followed this strategy, then those who said ‘black’ after gnome 1 must have been wearing a black hat that was seen by gnome 1. The number of black hats in front of every gnome from 2 to $n - 1$ toggles from odd to even and from even to odd every time one of them says ‘black.’ When we get to gnome $n - 1$, the number of black hats in front of him will be odd if the number times the word ‘black’ was heard is odd and even if that number is even. Since gnome n knows whether the number of black hats in front of him is odd or even, and he now also knows whether the number of hats remaining (including his own) is also odd or even, the same logic holds as for gnome 2 above.”

OTHER RESPONDERS

Responses have also been received from R. Bily, G. Blondin, B. Bramley, N. Cohen, D. Detlefs, G. Engelstein, A. Faller, J. Feil, M. Fineman, M. Fischler, R. Giovanniello, J. Grossman, J. Harmse, S. Kanter, D. Katz, J. Korba, L. Kyser, A. LaVergne, J. Lee, F. Marcoline, D. McMahon, K. Miller, T. Mita, H. Nahmias, B. Nalebuff, J. Oehrle, P. Ojanen, A. Ornstein, J. Prussing, T. Royappa, E. Sard, L. Schaider, E. Sheldon, E. Signorelli, T. Sim, T. Taylor, A. Ucko, R. Wake, and J. Wouk.

PROPOSER’S SOLUTION TO SPEED PROBLEM

One was born 12:01 A.M. on 1 January 2000 in London, England, and is thus nearly eight hours older than the other, who was born 11:59 P.M. on 31 December 1999 in San Diego, CA (or the equivalent in 2000/2001 if you believe millennia start in ‘01). ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.