I am pleased to report that our long time "Puzzle Corner" contributor, Avi Ornstein, has just released a new book. I wish him success with "Sonia in Vert."

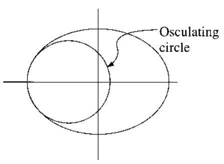
PROBLEMS

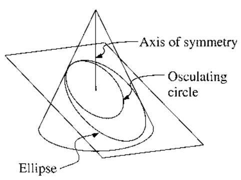
M/A 1. Larry Kells wants to know the highest contract South, the declarer, can make with the following distribution of the four hands. The unusual aspect is that the opponents are to cooperate in this venture, i.e. the hand is to be played against "worst defense".

	♠ 10 8 6 ♥ 9 ♦ 9 8 7 6 5 4 3 2 ♣ 9	
♠ A Q ♥ 8 ♦ K 10 ♣ A K Q 8 6 4		♠ A Q ♥ A K Q J 7 5 3 ♦ A Q J ♣ 10
	♠ 5 4 3 2 ♥ 10 6 4 2 ♦ – ♣ J 7 5 3 2	

M/A 2. Albert Mullin notes that 4159 is the first 4-digit prime to occur as consecutive digits of the decimal expansion of pi. He wonders what is the first 10-digit prime to occur this way.

M/A 3. Tim Barrows has sent us what looks to me to be a rather serious 3D geometry problem. He writes. An "osculating circle" is a circle which matches the slope and radius of curvature of another curve at some point. An osculating circle of an ellipse is shown in the diagram. In this example, the matching point is the intersection of the ellipse with its major axis. Let us call this a "major axis osculating circle." Consider now the case in which an ellipse is created from the intersection of a plane and a right circular cone. The orientation is such that a line drawn from the tip of the cone to the nearest point on the ellipse is perpendicular to the plane, i.e. the plane is at a right angle to the side of the cone at that point. In other words, the tilt of the plane is equal to the half angle of the cone. Show that the center of one of the major axis osculating circles of this ellipse lies on the axis of symmetry of the cone.





SPEED DEPARTMENT

A quickie from John Prussing. Two unmarked coins are in a box: a fair coin with probability of heads p and a "funny" coin with p equal to 0.3. One coin is selected at random and flipped 10 times, resulting in 4 heads and 6 tails. Which coin is more likely to generate this result?

SOLUTIONS

N/D 1. Robert Wake writes that it seems unlikely that N/S can get any unluckier than the 28-point hand shown below. At this range or above, there must be some contract where they can take 7 tricks if they get the chance, so E/W can only prevail if they can take 7 tricks off the top in all four suits and notrump. Seven tricks in notrump requires, at a minimum, either one long suit or (as below) two suits headed by at least the AQ. Relying on one long suit means trouble in suit contracts, because a suit that will run at notrump will pose too many complications for defeating contracts in the other three suits without enough high cards.

So this is the best pair of N/S hands I could find that is unable to make any contract from either side. Since the hands are totally symmetric, we can assume South is declarer without loss of generality. East-West take the club finesse, then the diamond finesse, then the second club finesse and East cashes one more high club. At clubs or notrump, East cashes the fourth club, plays a diamond, and they have eight tricks. Otherwise, East switches to diamonds, and the fourth diamond either is good (diamonds), is ruffed high by partner (hearts), or forces declarer to ruff and sets up West's long trump as the seventh trick (spades).

	♠ 9 8 ♥ A K Q J ♦ 7 6 5 4 ♣ K J 9	
♠ 10 4 3 2 ♥ 7 6 5 ♦ A Q 10 8 ♣ 3 2		♠ 7 6 5 ♥ 10 4 3 2 ♦ 3 2 ♣ A Q 10 8
	♠ 9 8 ♥ A K Q J ♦ K J 9 ♣ 7 6 5 4	

N/D 2. There seems to be a question of scaling, but most solutions agreed that the rate of new restaurants should be proportional to the square root of the "death rate." Ed Sheldon sent us a detailed solution, which appears on the Puzzle Corner website, cs.nyu.edu/~gottlieb/tr. Due to space limitations we present an abbreviated version here.

The problem is one of rates. Let us assume that the favorites die of at an average interval of N_d (measured in meals eaten out). The sampling of new restaurants must be sufficient to produce, on average, one new favorite over the same interval. If we measure the enjoyment (E) on a scale of 0 to 1, the enjoyment of a new restaurant will be assumed to be a random value from 0 to 1. It will also be assumed that the pool of new restaurants is unlimited. Let us now assume the favorites have a value of E_0 or higher. Since the distribution is linear, the average enjoyment value of the favorites will be $E_f = (1+E_0)/2$ and the average value of the rejects in the pool (values of less than E_0) is $E_p = E_0/2$.

Now on average, for every $1/(1-E_0)$ samples, there will be one sample above E_0, and this number of sampling visits must be taken in the interval N_d. Now for enjoyment purposes, one of the sampling visits was enjoyable, so the number of inferior members of the pool visited will be $N_a = E_0/(1\text{-}E_0)$ and the average enjoyment will be $E_{av} = [E_p \times N_a + E_f \times (N_d - N_a)] / N_d$. This can be simplified to

$$E_{av} = ½ \left[1 + E_0 - \frac{E_0}{N_d(1 - E_0)}\right].$$

The average value is thus a function of E_0 and N_d. Assuming the death rate is constant, the value of E_0 that will maximize average enjoyment can be found by differentiating, and setting the derivative equal to zero

$$2 \times E_{av} = 1 + E_0 - \frac{E_0}{N_d(1 - E_0)}$$

which simplifies to $E_0 = 1 - 1/\sqrt{N_d}$.

The problem asked for the fraction of the time you should try new restaurants, which is $[1/(1\text{-}E_0)]/N_d = 1/\sqrt{N_d}$.

Since N_d is the interval between deaths, or the reciprocal of the death rate, the fraction is $\sqrt{}$*Deathrate*.

N/D 3. This problem received several compliments from its solvers, several of whom submitted very fine work. Jay Sinnett made it look almost easy, which I firmly believe it was not. He writes.

Given thirteen stacks of 4 coins each, knowing that one stack has identical counterfeit coins that weigh less than standard coins (by an amount not exceeding 5 grams), and knowing that good coins all weigh an integral number of grams, how can we determine the following in two weighings: the weight of good coins; the stack with the counterfeit coins; the weight of the counterfeit coins?

The first part is easy: put more than 20 coins on the scale and weigh them. Divide the result by the number of coins. If the result (average weight of a coin) is not an integer, round it up to the next integer to find the weight each good coin. This works because even if all four counterfeit coins are in the group, they can only create a deficit of less than 20 grams, which when divided by a number larger than 20 must lower the apparent average weight of a coin by less than one gram.

To tackle the second and third questions, consider that in each weighing there can be zero, one, two, three, or four counterfeit coins – a total of five possibilities per weighing. This means there are a total of 25 possible outcomes for the two weighings. Let us define the "deficit" in each weighing as the difference between the ideal total weight of good coins and the actual total weight registered on the scale. I've shown in this table the ratio of the deficit from the first weighing divided by the deficit from the second weighing; examining the table we find that there are 13 unique ratios possible (and 12 duplicates).

Therefore, we can arrange coins from the different stacks in such a way that each stack can contribute deficits according to one of the unique ratios in the table. Then it becomes a simple matter to match up the deficit ratio we measured to the stack that caused it, and also a simple matter to calculate the missing weight in the counterfeit coins. In my solution, there will be 26 coins in each weighing. Space constraints do not permit us to show Sinnett's table. His entire solution can be found on the puzzle corner web page.

Counterfeit coins in First Weighing

Counterfeit coins in Second Weighing	0	1	2	3	4
0	NA	∞	∞	∞	∞
1	0	1	2	3	4
2	0	.5	1	1.5	2
3	0	.3333	.6666	1	1.3333
4	0	.25	.5	.75	1

BETTER LATE THAN NEVER

2009 J/A 2. Aaron Ucko reports that I dropped an n from the expression for the general minimum number of touches, which should have been.

N/D SD. I normally do not print comments on speed problems but must this time as the solution given was wrong. We forgot that 25 and 50 have two(!) factors of 5. As a result 52! ends in 12 zeros.

OTHER RESPONDERS

Responses have also been received from F. Albisu, S. Allen, D. Aucamp, J. Bobbitt, G. Borrmann, S. Brown, S. Brown, D. Carlton, G. Case, G. Chan, B. Chapp, S. Clarke, P. Cohen, M. Cohen, J. Cote, C. Dailey, D. Detlefs, D. Dewan, M. Eiger, D. Ertas, D. Ewing, L. Fattal, R. Fawcett, M. Fineman, D. Flanagan, D. Freeman, J. Freilch, R. Giovanniello, G. Goissiere, B. Gold, B. Jacobsen, E. Jensen, B. Julian, J. Karisson, J. Kenton, D. Kulp, A. Kunin, Richard Lamson, A. LaVergne, I. Lai, M. Lawler, B. Layton, M. Lehman, M. Liu, M. Lugo, T. MacDiarmid, J. Mahoney, R. Mandl, T. Maxwell, D. McIlroy, D. Mellinger, W. Meyer, L. Nissim, B. Nunes, S. Oh, T. Palmere, K. Rosato, A. Rosenfield, J. Russell, T. Sauke, P. Schottler, D. Seih, H. Shaw, D. Sidney, L. Siegel, D. Sieh, E. Signorelli, J. Steele, D. Stephenson, G. Stith, M. Strauss, I. Sturdy, A. Sutherland, K. Szolusha, K. Takase, T. Terwilliger, T. Tsakiris, T. Tu, M. Turpin, R. Utz, S. Vakil, G. Wassermann, B. Weggel, A. Wei, and F. Yee-Roth.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The fair coin is slightly more likely. The likelihood of 4H6T is $p^4(1 - p)^6$, which equals 9.52×10^{-4} for the funny coin and 9.76×10^{-4} for the fair one. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.