Since this is the first issue of an academic year, let me once again review the ground rules. In each issue I present three regular problems, the first of which is normally related to bridge (or chess or some other game), and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e., four months) later, one submitted solution is printed for each; I also list other readers who responded. For example, the current issue contains solutions to the regular problems posed in May/June.

The solutions to the problems in this issue will appear in the January/February column, which I will need to submit in early October. Please try to send your solutions early to ensure that they arrive before my deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that have remained unsolved.

### PROBLEMS

**S/O 1.** Our first problem this issue is from Jorgen Harmse, inspired by Frank Schuld's book *The Simple Squeeze in Bridge*. South is declarer at four spades.

| | | |
|---|---|---|
| | ♠ 6 5 4 3 2<br>♥ A K 4<br>♦ Q<br>♣ A 8 5 3 | |
| ♠ Q J 10 8 7<br>♥ Q 10 9 7 3<br>♦ A 8<br>♣ J | | ♠ –<br>♥ J 8<br>♦ 9 7 6 4 2<br>♣ Q 10 9 7 4 2 |
| | ♠ A K 9<br>♥ 6 5 2<br>♦ K J 10 5 3<br>♣ K 6 | |

How can he make the contract against a spade queen lead? Can he make it against a heart lead? In both cases assume double dummy (all hands visible, best play on both sides).

**S/O 2.** The following problem is from Ermanno Signorelli. Given a triangle ABC with a length of BC, one can construct a circle with center A and radius *a*. Similarly for B with radius *b* (the length of AC) and for C with radius *c* (the length of AB). How can one construct an exterior circle tangent to the other three?

**S/O 3.** Howard Cohen serves up the following. If a tennis player has probability *p* of winning each point on her serve, what is the probability that she will win her service game?

### SPEED DEPARTMENT

Ted Saito asks: When can 99 exceed 100? As a hint, I will add that I have used this solution for many years in everyday life.

### SOLUTIONS

**M/J 1.** Jorgen Harmse wants you to suppose that White has a forced mate (possibly exceeding the 50-move rule or other limits, but with no opportunity for Black to arrange a stalemate or to checkmate White) and plays to minimize the number of moves. How many times can the same position (arrangement of pieces on the board) *conceivably* occur? You need not show that an actual game can give rise to this scenario.

Several readers mentioned the threefold repetition rule, which is the limiting factor. However, the problem states that the position is simply the arrangement of pieces; for threefold repetition to occur, more than just the piece positions must be repeated, as Harmse's solution below indicates.

Each occurrence of the position must differ from the others in a way that influences future play but is not shown by the arrangement of the pieces. The possibilities are the move, (long-term) ability to castle, and ability to capture en passant. En passant capture is possible (if at all) only the first time the position occurs, and White's ability to castle cannot be the issue. Thus, the difference is in the move and Black's ability to castle. Of the four castling states for Black, only three can occur in any one game. Thus, the same arrangement of pieces can occur at most six times without a complete repetition.

To see that such repetition is conceivable, imagine that position A occurs for the first time with Black to move and before Black's king or either Black rook has moved. White may have a sequence that gains tempo, returning to position A with the move. White may then have an attack that Black can defeat only by castling. White therefore launches a different sequence, forcing Black to move a rook and reaching position A for the third time with Black to move. The same tempo-gaining sequence as before allows White to gain the move for the fourth occurrence of position A. Yet another threat forces Black to move the other rook or the king (or both, perhaps even by castling), and the position occurs for the fifth time, with Black to move. White then employs the tempo-gaining sequence for the third time to effect a sixth occurrence of position A, and then launches the mating attack.

**M/J 2.** Rocco Giovanniello has four balls: one of radius 1, the second of radius 2, the third of radius 3, and the fourth of radius 4. The three largest balls are resting on a level surface and are tangent to each other. Now place the smallest ball in the crater between the other three. How far is this ball off the ground?

James Simmons used a favorable placement of the balls (i.e., a cleverly chosen coördinate system) to obtain six equations in six unknowns that could be solved fairly easily. His solution follows. Let $B_4$, $B_3$, $B_2$, and $B$ denote the balls of respective radii 4, 3, 2, and 1. In a three-dimensional, right-handed Cartesian coördinate system, the centers of the balls, with no loss of generality, may be taken as $C_4 = (0, 0, 0)$, $C_3 = (x_3, 0, -1)$, $C_4 = (x_2, y_2, -2)$, and $C = (x, y, z)$ so that $B_4$, $B_3$, and $B_2$ lie on the horizontal plane $z = -4$ and $B$ lies a distance $z + 3$ above this plane.

There are six unknowns $x_3$, $x_2$, $y_2$, $x$, $y$, and $z$, and six known distances $|C_4C_3|$, $|C_4C_2|$, ... , $|C_2C|$ that we can use to determine these unknowns. Thus,

$$|C_4C_3|^2 = x_3^2 + (-1)^2 = 7^2$$
$$\Rightarrow x_3 = 4\sqrt{3}$$

$$|C_4C_2|^2 = x_2^2 + y_2^2 + (-2)^2 = 6^2$$
$$\Rightarrow x_2^2 + y_2^2 = 32$$

$$|C_3C_2|^2 = (x_2 - x_3)^2 + y_2^2 + (-2 + 1)^2 = 5^2$$
$$\Rightarrow x_2 = 7/\sqrt{3}, y^2 = \sqrt{47/3}$$

$$|C_4C|^2 = x^2 + y^2 + z^2 = 5^2$$

$$|C_3C|^2 = (x - x_3)^2 + y^2 + (z + 1)^2 = 4^2$$
$$\Rightarrow x = (z + 29)/4\sqrt{3}$$

$$|C_2C|^2 = (x - x_2)^2 + (y - y_2)^2 + (z + 2)^2 = 3^2$$
$$\Rightarrow y = \tfrac{1}{12}\sqrt{\tfrac{3}{47}}(17z + 109)$$

Substituting the last two equations into the fourth gives

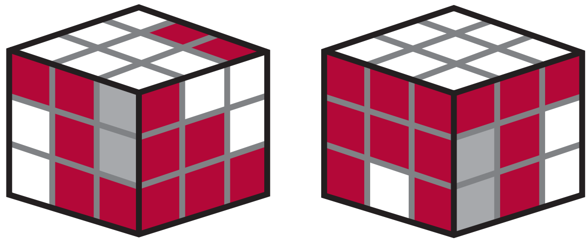
$$\frac{(z + 29)^2}{48} + \frac{(17z + 109)^2}{48(47)} + z^2 = 25$$

or

$$54z^2 + 134z - 104 = 0$$

Solving the quadratic for $z$, taking the larger root, and adding 3, we find that the distance of B above the plane is 3. 6208 ...

**M/J 3.** The MIT logo reminds David Hagen of a Rubik's Cube with recolored faces. However, his cube has been scrambled. He asks you to restore the logo in the minimum number of steps. The original cube and the desired result are below (the *I* in *MIT* is lowercase gray).



I learned from Lucas Garron, a self-described "cuber," that the website www.speedsolving.com/wiki/index.php/notation specifies the "standard notation" for Rubik's Cube moves. Using this notation, Garron's solution is L' B' L', followed by the rotation x2 y to orient the cube. He writes, "If we imagine the desired-state picture rotated 180°, we can see that most of the colored stickers would remain the same." In case that is not clear, Garron supplied a very cute animation that I recommend to you all: see archive.garron.us/solves/2009/MIT_puzzle.htm. You can rotate the cube by mouse-dragging just outside it; animate the solution above by using the controls along the bottom of the Web page.

Finally, Garron shows us how to get the MIT logo from an initial cube, both one that has just three colors:

alg.garron.us/?alg=F2_U_B2_D_U-_F-_L2_F-_L2_F2_L2_F2_D-_F2&name=%22MIT_Logo%22&scheme=rrwxxw

and the standard one with six colors:
alg.garron.us/?alg=F2_U_B2_D_U-_F-_L2_F-_L2_F2_L2_F2_D-_F2&name=%22MIT_Logo%22.

## BETTER LATE THAN NEVER

**J/F 3.** Robert Ackerberg defines *P* as the point where the extended side of the square hits the bottom of the rectangle. He notes that by Pythagoras's theorem, the distance from *P* to the right side is $\sqrt{1 - (1/x)^2}$.

## OTHER RESPONDERS

Responses have also been received from F. Albisu, C. Antonini, E. Bade, R. Bird, C. Brown, L. Cangahuala, P. Cassady, J. Chandler, G. Coram, A. Day, A. Feltman, R. Giovanniello, J. Hardis, T. Harriman, C. Hibbert, B. Ho, R. Moeser, W. Peirce, J. Prussing, S. Rodriguez, K. Rosato, J. Ross, E. Sard, R. Schargel, M. Seidel, S. Silverberg, A. Taylor, and T. Terwilliger.

## PROPOSER'S SOLUTION TO SPEED PROBLEM

When entered as cooking time on a microwave oven. ■

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Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.