This being the first issue of a calendar year, we again offer a "yearly problem," in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 9) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2008 yearly problem is in the "Solutions" section.

PROBLEMS

Y2009. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 9 exactly once each; the operators +, -, × (multiplication), and / (division); and exponentiation? We desire solutions with the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 9 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator. Zero to the zero power is *not* permitted. For some reason I can't recall, I permit 00 to be used for zero (no operator present) but do *not* permit other number with leading zeroes (e.g., 09).

J/F 1. For a little variety, Eugene Sard offers us a different kind of bridge problem. "Five bridge players, A, B, C, D, and E, want to rotate partnerships with A out first, B out second, etc., until E is out last, so that no partnership is repeated. This must be a common problem that many players have presumably worked out, but the question here is how many solutions exist, and what they are."

J/F 2. William Tucker's goal in life (I suppose) is to solve every possible crossword puzzle. Step one is to determine how many there are. He is looking for 15-by-15 grids of "open" and "blacked out" squares obeying the standard 180° rotational symmetry and with no words of length one or two (all open squares are contained in a row or column of at least three open squares). No row or column can be completely blacked out.

J/F 3. Steve Jones believes that given any two rectangles of equal area, either rectangle can be cut into pieces that can be reassembled into the other rectangle. Is he right?

SPEED DEPARTMENT

Philip Lally wants you to move one digit to correct the equation $101 - 102 = 1$.

SOLUTIONS

Y2008. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 8 exactly once each; the operators +, -, × (multiplication), and / (division); and exponentiation? We desire solutions containing the minimum number of operators, and among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 8 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator. Zero to the zero power is not permitted.

The following solution is from Glen Stith. An asterisk indicates that the digits are used in the preferred order.

*$1 = 2^0 - 0^8$	$21 = 20 + 8^0$
*$2 = 2 - 00^8$	*$25 = 200 / 8$
$3 = 2 + 8^{00}$	*$28 = 20 + 0 + 8$
$4 = 00 + 8/2$	$40 = 0 + 80 / 2$
$6 = 00 - 2 + 8$	$60 = 80 - 20$
$7 = 8 - 2^{00}$	$64 = 8^2 - 00$
*$8 = 2 \times 00 + 8$	$78 = 80 - 2 - 0$
*$9 = 2^{00} + 8$	$79 = 80 - 2^0$
*$10 = 2 - 00 + 8$	$80 = 80 - 0^2$
*$12 = 20 - 0 - 8$	$81 = 80 + 2^0$
*$16 = (2 + 00) \times 8$	$82 = 80 + 2 + 0$
$19 = 20 - 8^0$	$100 = 80 + 20$
*$20 = 20 - 0^8$	

S/O 1. Bridge books say that the success of the defense often depends on the opening lead. Larry Kells wants to know the greatest difference the opening lead can make in the final number of tricks taken by the defense in a suit contract. You are to specify all four hands and should assume best play on all sides, except for the opening lead.

Were no-trump allowed, the answer would have been 13 tricks (i.e., declarer taking all tricks vs. taking none); however, for a suit contract it is more difficult. Frank Model found a cross-ruff-based solution with a swing of eight tricks (declarer taking 13 vs. taking 5). The proposer found a cute improvement (predicted by Model) of one trick with the following hand:

North:
♠ 2
♥ A K Q J 10
♦ A K Q J 10
♣ 4 3

West:
♠ 10 9 8 7
♥ 5 4 3 2
♦ –
♣ A K Q J 10

East:
♠ 6 5 4 3
♥ –
♦ 5 4 3 2
♣ 9 8 7 6 5

South:
♠ A K Q J
♥ 9 8 7 6
♦ 9 8 7 6
♣ 2

If West leads a trump against seven spades, declarer draws the trumps and easily makes the contract. If West makes any other lead, the defenders take the first nine tricks via a cross-ruff in the red suits and the ace of clubs.

S/O 2. Perhaps to compensate for our favoring bridge over go puzzles, Robert High has sent us three *independent* cryptarithmetic puzzles celebrating the Eastern game. In these problems you are to substitute a digit for each letter so that the equation relating the numbers is valid (we use * to indicate multiplication). For each puzzle, no two letters represent the same digit. Finally, no number begins with zero. The three problems are

GO * GO = GAME
GO * GAME = BEAUTY
GO * BOARD = STONES

The consensus seems to be that the first one can be "figured out" but that the other two need a computer attack. The following solution is from John Chandler.

"The first problem is simple. The square of a two-digit number has to have the same leading digit, so that digit has to be 9 or just possibly 8, and the second digit has to be large. The obvious first guess at the two-digit number, 98, turns out to be the answer:

GO * GO = GAME – 98 * 98 = 9,604

"The second problem is so much less constrained (the leading digit has to be 3 or more) that I finally succumbed after many years to the temptation to write a computer program to solve such problems:

GO * GAME = BEAUTY - 34 * 3,762 = 127,908
GO * BOARD = STONES - 12 * 52,708 = 632,496"

S/O 3. In discussions about sun exposure, one rule of thumb is that the chances of sunburn are greatest when the sun is more than 45° above the horizon. At what latitude and at what time of the year is the sun above 45° for the longest continuous time period? I suppose that Larry Kells asks us this question so that he can decide when to schedule a prolonged outdoor bridge game.

The solution below is from Ed Sheldon. Jonathan Smith notes that the location in question passes through the southern United States, with Atlanta, Phoenix, and Los Angeles pretty close. Nonetheless, he believes that the "best" sunburn can be had closer to the equator, where the increased intensity of the solar radiation easily overcomes any reduction in the time that the sun is high in the sky.

"This will be treated as a simple three-dimensional geometry problem. There are four angles to consider: q = the rotational angle of the earth, with 0 at high noon; O = the altitude of the sun with respect to the equator; $\tilde{a}$ = the latitude of interest; and $\acute{a}$ = the angle of the sun with respect to local vertical (= 45°). At any point in time q, the angle $\acute{a}$ can be found by the dot product of the unit vectors representing the point on the earth and the sun.

$$P_{\text{earth}} = i\cos(\tilde{a})\cos(q) + j\cos(\tilde{a})\sin(q) + k\sin(\tilde{a})$$

$$P_{\text{sun}} = i\cos(O) + j\,0 + k\sin(O)$$

$$\cos(\acute{a}) = \cos(\tilde{a})\cos(q)\cos(O) + 0 + \sin(\tilde{a})\sin(O)$$

from which

$$\cos(q) = [\cos(\acute{a}) - \sin(\tilde{a})\sin(O)] / [\cos(\tilde{a})\cos(O)]$$

The maximum angle occurs when the cosine is minimum. The angle $\acute{a}$ is fixed, and for any value of O, the minimum can be found by differentiating with respect to $\tilde{a}$ and setting the result to 0.

$$d\cos(q)/d\tilde{a}=\{[\cos(\tilde{a})\cos(O)][-\cos(\tilde{a})\sin(O)]-[\cos(\acute{a})-\sin(\tilde{a})\sin(O)][-\sin(\tilde{a})\cos(O)]\} / [\cos(\tilde{a})\cos(O)]^2$$

$$0=-\cos^2(\tilde{a})\cos(O)\sin(O)-[-\cos(\acute{a})\sin(\tilde{a})\cos(O)+\sin^2(\tilde{a})\sin(O)\cos(O)]$$

$$0=-\cos^2(\tilde{a})\cos(O)\sin(O)+\cos(\acute{a})\sin(\tilde{a})\cos(O)-\sin^2(\tilde{a})\sin(O)\cos(O)$$

$$0=\cos(\acute{a})\sin(\tilde{a})\cos(O)-\cos(O)\sin(O)$$

$$0=\cos(\acute{a})\sin(\tilde{a})-\sin(O)$$

$$\sin(\tilde{a})=\sin(O)/\cos(\acute{a})$$

The higher the sun, the greater the time. If the sun were at 45° or above, the pole would have 24 hours of the specified exposure. The sun's maximum altitude (O) is about 23.45°. So the latitude of greatest exposure is arcsin(sin(23.45°)/cos(45°)) = 34.25°. The time of year is the first day of summer—June in one hemisphere, December in the other. At that time of year and latitude, the sun is at 45° when the angle q is 50.42°, which multiplied by (2*24/360) is 6.72 hours.

BETTER LATE THAN NEVER

2008 M/A 3. Donald Aucamp believes that the extended problem is considerably more complicated than our solution suggests. His detailed analysis shows that if the bug begins on the left end face at a = b = x and ends on the right end face at c = d = x, where x = .366025, the distance traveled will be 3.0119442.

M/J 1. Several readers had trouble with the published solution, which is not surprising, since I mistakenly omitted a sentence. A spade must be ruffed in dummy before a club can be led to the king. Sorry for my error.

OTHER RESPONDERS

Responses have also been received from A. Avakian, C. Dale, D. Dechman, J. Feil, S. Feldman, M. Fitzgibbons, R. Giovanniello, J. Harmse, T. Harriman, R. Hess, J. Karnofsky, L. Kyser, B. Layton, Y. Li, Z. Moledina, E. Nelson-Melby, A. Ornstein, D. Plass, L. Pringle, S. Resnikoff, K. Rosato, E. Signorelli, B. Sutton, A. Taylor, A. Ucko, S. Ulens, G. Virgile, and R. Wake.

PROPOSER'S SOLUTION TO SPEED PROBLEM

$101 - 10^2 = 1$ ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.