It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

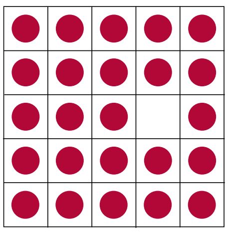
As responses arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred, as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent by e-mail, since these produce fewer typesetting errors.

PROBLEMS

N/D 1. Yet another bizarre deal occurred at Larry Kells's duplicate-bridge club. At one table, North-South bid and made seven spades. At another table, on the same deal, East-West bid and made one spade redoubled! Now, surely if one side can make a grand slam in a suit, the other side can't possibly make any contract in that same suit ... can they? Some defender must have made a terrible mistake in play. However, when the scorer asked for verification of the scores, all the players at the two tables involved confirmed that those contracts were made, and furthermore, that no defender ever made any error! Unfortunately, the cards were all mixed up after they were played for the last time. Can you help reconstruct the deal?

N/D 2. Loren Bonderson enjoyed the problem of finding the grazing area of a goat tied to a silo so much that he extended it to three dimensions (and moved it from farming to astronautics). If an astronaut is tethered to a spherical satellite of radius R with a tether of length πR, what is the volume of space the astronaut may reach?

N/D 3. Perhaps to balance the increase of dimensions in the previous problem, Rocco Giovanniello has lowered his 3-D "wink problem" to a mere two dimensions.



Consider a five-by-five checkerboard with 24 of the squares containing a wink each; the remaining square is empty. Using up-down and left-right jumps, can you remove winks until only one remains? The specific problem posed uses the starting configuration at right and permits the one remaining wink to be on any square.

One could also consider all 25 possible starting positions and all 25 possible ending positions and ask which of these 625 games can be solved. By symmetry, the number of games can be reduced considerably. But we are not asking for a solution to that problem.

SPEED DEPARTMENT

Robert Ackerberg wants you to imagine that the earth is a perfect sphere with radius 4,000 miles and that a string is surrounding the earth at the equator, stretched tight to the ground. How much more string would be needed if we wanted the cord to be uniformly two inches above the equator?

SOLUTIONS

J/A 1. A max-min problem from Larry Kells. What is the smallest number of high-card points you can have and be sure of defeating all small slams, and what is the largest number of high-card points you can have and not be sure of defeating all small slams? Assume best play on both sides.

Zane Moledina found the lowest value, this 18-point hand:

♠ J 10 9 8 7
♥ Q J 10 9
♦ A K
♣ A K

He notes that in any trump contract, two winners are assured. In no-trump, once a minor suit is led, four tricks are immediate. If the major-suit top honors are taken first, the carnage is great.

For the best possibly unsuccessful hand we turn to Tom Terwilliger's 35-point example:

♠ A Q
♥ A K Q
♦ A K Q J
♣ A K Q J

Tom notes that the hand sitting after this can be

♠ K J xxxx
♥ xxxxxxx

and the hand before it could be the other five spades with no hearts and any club/diamond holding. He describes the play as follows:

"My best lead is a club/diamond. Declarer trumps, trumps a heart, leads a spade through my AQ. I can win or not; it doesn't matter. If I win and play another club/diamond, declarer trumps, trumps a heart, plays a trump to the K♠, trumps a third heart, and trumps another club/diamond. Now the rest of his hearts are good, and declarer has a trump to spare. If I duck the first trump lead, then the order of tricks is merely shuffled."

J/A 2. Jerry Grossman has equipped n children with loaded water pistols and has them standing in an open field with no three of them in a straight line, so that the distances between pairs of them are distinct. At a given signal, each child shoots the child closest to him or her with water. Show that if n is any even number, then it is possible (but not necessarily the case) that every child gets wet. Show that if n is odd, then necessarily at least one child stays dry.

Timothy Chow notes that this problem appears on page four of Peter Winkler's book *Mathematical Puzzles* in the guise of "soldiers in the field." Winker attributes the problem to the sixth All Soviet Union Mathematical Competition in Voronezh in 1966.

Sameer Shah appears to be quite expert with water pistols. Indeed, so was I, a *long* time ago. Shah writes,

"Let us tackle the second problem first. If n is odd, we prove via contradiction that one child remains dry. Assume that all the children get soaked. If we visualize the children on a plane, and draw an arrow from the child shooting to the child being shot, we notice that for every child to get soaked, 'arrow cycles' must form. That's because every child must be shot exactly once (otherwise there would be a child shot twice, which leads to the conclusion that there is a child who is not getting shot). So we have either cycles of two children shooting each other or bigger cycles of three or more children shooting each other. But the latter is impossible! In any group of three or more, there will always be two who are closest to each other, which means they soak each other. And since no one else can soak them (since no one can be shot more than once), you cannot form a cycle of three or more. Hence, we must have arrow cycles of just pairs of children. But there is the contradiction. We said that n was odd, so the children cannot all be in pairs! Hence our original assumption was incorrect, and there is at least one child who doesn't get soaked.

"To show the easier case, when n is even, we use the same analysis as above. We saw that if all the children were in pairs who shot each other, then everyone would get wet. This is possible by placing pairs of children very near each other but far from all other pairs. So if n is even, it is possible! But only if the children shoot in pairs. In all other cases, the same analysis we did for the odd number of children shows that at least one child remains dry."

To expand on this last point, David Detlefs first places three of the children in an equilateral triangle and a fourth at the triangle's center. Then the three vertex children will soak the center child and the center child can soak only one of those three. If these four children are far from the other children, we see that no other child soaks any of the four, so two remain dry. Finally, perturb the triangle slightly so that all distances are unique.

J/A 3. We close with another "logical hat" problem from Richard Hess. Each of logicians A, B, and C wears a hat with a positive integer on it. The number on one hat is the sum of the numbers on the other two. They make statements as follows:

A: "I don't know my number."

B: "My number is 15."

What numbers are on A and C?

William Seaman put on his thinking cap and derived A = 10 and C = 5. His key observation is the following: the situation in which A does not know his own number, but B knows his after hearing A's negative response, occurs if and only if either (1) A and C have the same number or (2) A's number is twice C's, and B's number is three times C's.

Proof: Suppose A and C have the same number, x. Then B's number is $2x$. Clearly, A knows only that his number is x or $3x$. B will see both x's and know that his own number is $2x$.

Suppose C's number is x, A's number is $2x$, and B's number is $3x$. A knows only that his number is $2x$ or $4x$. B knows that his number is $3x$ or x, but can eliminate x, since in that case, A would see two x's and know that his own number was $2x$.

The above proves that in (1) and (2), we have a negative response from A and a positive response from B. To show that these are the only cases that produce these two responses, we consider the three possibilities where C, then A, then B is the sum of the other two.

(i) C's number is x, A's number is $x + y$, and B's number is y. If $y = x$, then A knows that his number is $2x$ and responds positively. If $y \neq x$, A responds negatively. B knows only that his number is y or $2x + y$. Since neither of these numbers equals x, both are consistent with A's negative response, and B does not know his number.

(ii) C's number is $x + y$, A's number is x, and B's number is y. A responds negatively. B knows that his number is y or $2x + y$. Neither number equals $x + y$, so B can eliminate neither and cannot know his number.

(iii) C's number is x, A's number is y, B's number is $x + y$, and $x \neq y$ ($x = y$ is (1), above).

(iiia) $x > y$. A responds negatively. B knows that his number is $x + y$ or $x - y$. Neither $x + y$ nor $x - y$ equals x, so B can eliminate neither.

(iiib) $x < y$. A responds negatively. B knows that his number is $x + y$ or $y - x$. Since $x + y \neq x$, $x + y$ cannot be eliminated. However, $y - x$ can be eliminated when $y - x = x$; i.e., when $y = 2x$, which is (2), above, proving the claim.

In the special case of J/A 3, we know that (1) is impossible since B's response was an odd number. Thus, (2) must hold, with $3x = 15$.

BETTER LATE THAN NEVER

M/A 3. Chatchawin Charoen-Rajapark points out, to my embarrassment, that the same problem appeared with its solution in the January/February 2000 "Puzzle Corner."

OTHER RESPONDERS

Responses have also been received from F. Aibisu, R. Canales, L. Casey, T. Chow, D. Ertas, J. Feil, M. Fineman, J. Freilich, P. Horvitz, S. Howlett, D. Katz, P. Kramer, J.-W. Maessen, R. Merrifield, J. Mohr, A. Ornstein, S. Portnoy, M. Power, J. Prussing, K. Rosato, H. Shaw, S. Stith, R. Whitman, K. Zeger, and J. Zissu.

PROPOSER'S SOLUTION TO SPEED PROBLEM

About one foot! The circumference of a circle is $2\pi R$. Hence the new circumference exceeds the old by $2\pi \times 2" \approx 12.5"$. ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.