This is the first issue of a new academic year, so I'll review the ground rules. Each issue, I present three regular problems—the first normally related to bridge (or chess or another game)—and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns (four months) later, one submitted solution is printed for each; I also list other readers who responded. For example, the current issue contains solutions to regular problems posed in May/June.

I'm writing this column in mid-June, so the column containing the solutions should be due in mid-October. Please send solutions early, so they arrive before my deadline. Late solutions, and comments on published solutions, are acknowledged in later issues in the "Other Respondents" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that remained unsolved when first presented.

Four MIT undergraduate puzzle champions have developed a Mystery Hunt–style puzzle challenge at the request of the Alumni Association. The contest will feature lots of great prizes, including a grand prize of $1,000 in cash. Find out more and sign up to compete at alum.mit.edu/puzzle.

Robert Rothstein expands on my story from *the* Oliver Smoot about the Harvard Bridge. Rothstein writes, "I recall a discussion that took place on the pages of the *Tech* during my undergraduate days after a letter writer proposed that the bridge be renamed the MIT Bridge. The last word belonged to the editorial writer, whose argument I paraphrase from memory: we really don't want the good name of MIT associated with such a poorly designed bridge—and the name Harvard Bridge wouldn't be appropriate, since any fool knows that there is no bridge between Harvard and reality."

## PROBLEMS

**S/O 1**. Bridge books say that the success of the defense often depends on the opening lead. Larry Kells wants to know the greatest difference the opening lead can make in the final number of tricks taken by the defense in a suit contract. You are to specify all four hands and should assume best play on all sides, except for the opening lead.

**S/O 2**. Perhaps to compensate for our favoring bridge over Go puzzles, Robert High has sent us three independent cryptarithmetic puzzles celebrating the Eastern game. In these problems you are to substitute a digit for each letter so that the equation relating the numbers is valid (we use "*" to indicate multiplication). For each puzzle no two letters represent the same digit. Finally, no number begins with zero. The three problems are

GO * GO = GAME
GO * GAME = BEAUTY
GO * BOARD = STONES

**S/O 3**. In discussions about sun exposure, one rule of thumb is that the chances of sunburn are greatest when the sun is more than 45 degrees above the horizon. At what latitude and at what time of the year is the sun above 45 degrees for the longest continuous time period? I suppose that Larry Kells asks us this question so that he can decide when to schedule a prolonged, outdoor bridge game.

## SPEED DEPARTMENT

Mark Astolfi notes that second, third, fourth, etc., cousins can marry in all 50 states. However, third cousins Alex and Zelda, who could marry were they unrelated, cannot marry. Why not?

## SOLUTIONS

**M/J 1**. Larry Kells wants you to adapt my Pollyanna-like view of life to the world of bridge: "As a result of a bidding misunderstanding, you have arrived in seven hearts with the following hands. Your mission is to find a line of play that gives you a chance [albeit small —Ed.] of making it (against best defense), after the lead of a spade."

| North | South |
|-------|-------|
| ♠ 6 5 3 | ♠ A 7 4 |
| ♥ K 4 2 | ♥ A J 9 3 |
| ♦ A J | ♦ K 8 4 2 |
| ♣ A 9 5 4 2 | ♣ K 3 |

All solvers agree that the high cards should be with West, in a 5-2-3-3 distribution. Richard Hess offers the following specific deal and play:

|  | North |  |
|--|-------|--|
|  | ♠ 6 5 3 |  |
|  | ♥ K 4 2 |  |
|  | ♦ A J |  |
|  | ♣ A 9 5 4 2 |  |
| West |  | East |
| ♠ K Q J 10 9 |  | ♠ 8 2 |
| ♥ Q 10 |  | ♥ 8 7 6 5 |
| ♦ Q 10 9 |  | ♦ 7 6 5 3 |
| ♣ Q J 10 |  | ♣ 8 7 6 |
|  | South |  |
|  | ♠ A 7 4 |  |
|  | ♥ A J 9 3 |  |
|  | ♦ K 8 4 2 |  |
|  | ♣ K 3 |  |

First, South takes the spade ace, diamond finesse, and diamond ace. Then she draws two rounds of trump, ending in her hand, and cashes the king and eight of diamonds, discarding a spade from dummy. Finally, a club to the king and two heart tricks squeezes West in the black suits, and declarer makes the improbable seven-heart contract.

**M/J 2.** Edwin Field has a tetrahedron in which all six edges are perfect one-ohm resistors and the faces and interior have infinite resistance. What is the resistance measured between any two vertices?

The key, as several readers noted, is that, by symmetry, one of the edges will carry no current (equivalently, has equal potential at each endpoint). The following is from Glen Stith:

"It has been 48 years since I took 6.01 under Dr. Amar G. Bose, whom I consider the best lecturer I ever had. Even though I was a Course XVIII major, I think I remember how Dr. Bose taught me to solve this problem.

"Label the four vertices of the tetrahedron as points $A$, $B$, $C$, and $D$. The objective is to determine the equivalent resistance between any two vertices, say $A$ and $B$, when there is one ohm resistance along each edge and infinite resistance elsewhere. The applicable equation is Ohm's law, $I = V/R$, which says current equals voltage divided by resistance.

"If a voltage $V$ is applied across two vertices, say $A$ and $B$, then the current that flows between $A$ and $B$ will be the sum of the currents through all possible paths. However, from symmetry, the net current through the edge $CD$ will be zero. Therefore, the current will flow through paths $AB$, $ACB$, and $ADB$. The resistance $R_{ab}$ in path $AB$ is one ohm and the resistances $R_{acb}$ and $R_{adb}$ in the other two paths are each two ohms. Hence, the net current will be

$I = V/R_{ab} + V/R_{acb} + V/R_{adb} = V \times (1/1 + 1/2 + 1/2) = V \times 2 = V/(1/2)$
So the effective resistance between any two vertices is ½ ohm."

R. Ellis was not content with just the tetrahedron and furnished a table along with solution techniques for all the Platonic solids. For the larger solids, the nodes between which the resistance is to be calculated may not be adjacent; the "Node Spacing" column gives the distance between those nodes. Due to space limitations, the table has been moved to the "Puzzle Corner" website, cs.nyu.edu/~gottlieb/tr.

**M/J 3.** Donald Aucamp and Joyce Sabine offer the following method for checking the multiplication of positive integers. Reduce a (positive) integer $X$ to $D(X)$ by adding all the digits of $X$ and repeating the process until a single digit results. Then a requirement for $A \times B = C$ is that $D(D(A) \times D(B)) = D(C)$).

As an example, let's check if $6,843 \times 401 = 2,744,043$.

$D(A)$: $6,843 \rightarrow 6 + 8 + 4 + 3 = 21 \rightarrow 2 + 1 = 3$
$D(B)$: $401 \rightarrow 4 + 0 + 1 = 5$
$D(C)$: $2,744,042 \rightarrow 2 + 7 + 4 + 4 + 0 + 4 + 3 = 24 \rightarrow 2 + 4 = 6$
$D(D(A) \times D(B))$: $3 \times 5 = 15 \rightarrow 1 + 5 = 6$

Show that this procedure always works, or give an example where it fails.

All responders agree that the procedure always works and that it is related to "modulo-nine arithmetic." Many recalled that this procedure used to be called casting out nines. However, the procedure is not exactly the same as reducing the number mod 9 (e.g., 54 mod 9 is 0, and the procedure yields $5 + 4 = 9$, not 0). It is easy to see that if one or more of $A$, $B$, $C$ is zero, the formula holds trivially (both sides become zero). It is also easy to see that if none of them is zero, then applying $D$ never gives zero, and the following argument from Robert Wake shows that in this nontrivial case, the formula again holds.

"Short answer: Because $D(N)$ measures congruence mod 9 (more generally, in base $B$ it measures congruence mod $B$-1). Long answer: Because 10 is congruent to 1, each digit times 10 raised to its place is congruent to that digit, so when you add all these up for any $N$, you see that $N$ is always congruent to the sum of its digits, which by induction is congruent to $D(N)$ ('congruent' always means mod 9). Which means that $D(A^*B)$ is congruent to $A^*B$, which in turn is congruent to $D(A)^*D(B)$, and that in turn is congruent to $D(D(A)^*D(B))$. Thus, if $A^*B = C$, then $D(D(A)^*D(B))$ must be congruent to $D(C)$. Since they're both single-digit numbers, they can only be congruent mod 9 if they're the same (recall that neither is zero)."

Both my former NEC colleague Warren Smith and Jorgen Harmse discuss a related technique where one takes the alternating sum of the digits from right to left (for example, 6,843 yields $3 - 4 + 8 - 6 = 1$).

Steve Golson refers us to mathworld.wolfram.com/CastingOutNines.html.

### BETTER LATE THAN NEVER

**2007 N/D2.** See brad.edelman.googlepages.com/all.html for Brad Edelman's cool animations and alternative solutions.

### OTHER RESPONDERS

Responses have also been received from M. Brill, C. Brown, S. Busansky, G. Case, N. Cohen, G. Coram, J. Craig, E. Field, R. Giovanniello, S. Golson, J. Harmse, J. Karlsson, D. Katz, J. Kenton, J. Kesselman, P. Kramer, M. Lively, T. Mita, J. Pinson, E. Sard, S. Scheinberg, J. Steele, M. Teague, and D. Zalkin.

### PROPOSER'S SOLUTION TO SPEED PROBLEM

They are also siblings, as their parents are second cousins. ∎