Lorraine Mullin notes that Easter was quite early this year (23 March). The last time it was earlier was in the middle of the 19th century, and the next time will be 2285. I believe that in days gone by, calculating the date of Easter was quite a challenge.

PROBLEMS

J/A 1. A max-min problem from Larry Kells. What is the fewest high-card points you can have and be sure of defeating all small slams, and what is the most high-card points you can have and not be sure of defeating all small slams? Assume best play on both sides.

J/A 2. Jerry Grossman has equipped *n* children with loaded water pistols and has them standing in an open field with no three of them in a straight line, such that the distances between pairs of them are distinct. At a given signal, each child shoots the child closest to him or her with water. Show that if *n* is any even number, then it is possible (but not necessarily the case) that every child gets wet. Show that if *n* is odd, then necessarily at least one child stays dry.

J/A 3. We close with another "logical hat" problem from Richard Hess. Brian Benson refers readers interested in these problems to arXiv.org/abs/0710.2685. Recall that in logical hat problems, each logician is error free in his or her reasoning and knows that the same holds true for the others.

Each of logicians A, B, and C wears a hat with a positive integer on it. The number on one hat is the sum of the numbers on the other two. The logicians take turns making statements, as follows:

A: "I don't know my number."

B: "My number is 15."

What numbers are on A and C?

SPEED DEPARTMENT

Walter Cluett's vision is better than mine, but he can still see geometric shapes to only finite precision. He asks, "An equilateral triangle is to 20.7856 and a square is to 16 as a circle is to what?"

SOLUTIONS

M/A 1. Ira Rosenholtz wants you to find a legal chess position in which white has a king, a queen, the first move, and 31 legal moves; black has a king, a rook, a bishop, and a pawn on its original square of a7; and yet black wins with best play on both sides.

Todd Chase set up the following initial position:

White: K@g7 (8 legal moves); Q@d4 (23 legal moves). Total legal moves = 31.

Black: K@a1, R@c3, B@b2, p@a7 (per problem stipulation).

Chase offers the following analysis of the best play on both sides.

1. Kf8 Rc8+ winning
1. Kg8 Rc8+ winning
1. Kh8 Rc8+ winning
1. Kh7 Rc7+ winning
1. Kh6 Rc6+ winning
1. Kg6 Rc6+ winning
1. Kf6 Rc4 winning
1 . Kf7 Rc7+ winning
1. Qc4 Rxc4+ winning
1. Qb4 Rb3+ winning
1. Qa4+ Ra3+ winning
1. Qc5 Rxc5+ winning
1. Qb6 axb6 winning
1. Qxa7+ Ra3+ winning
1. Qd5 Rc5+ winning
1. Qd6 Rc6+ winning
1. Qd7 Rc7+ winning
1. Qd8 Rc8+ winning
1. Qe5 Rc7+ winning
1. Qf6 Rc6 winning
1. Qe4 Rc4+ winning
1. Qf4 Rc4+ winning
1. Qg4 Rc4+ winning
1. Qh4 Rc4+ winning
1. Qe3 Rxe3+ winning
1. Qf2 Rc2+ winning
1. Qg1+ Rc1+ winning
1. Qd3 Rxd3+ winning
1. Qd2 Rc2+ winning
1. Qd1+ Rc1+ winning

1. Qxc3 Bxc3+; 2. Kf7 a5; 3. Ke6 a4; 4. Kd5 a3; 5. Kc4 Kb2 winning

M/A 2. Ermanno Signorelli offers a problem that reminds me of the time I fell asleep on the commuter train, didn't notice that it had reached the terminal, and awoke in an empty car with all the doors locked.

Having fallen asleep at a concert, a man finds that he is locked within an auditorium that has five doors. Each door has two or more locks. He looks around and finds five key rings marked "Auditorium" hanging from hooks on a wall backstage. The set of rings holds all the keys, without duplication, to all the locks on all the doors, but the locks and doors are not identified on the keys. All locks and keys are unique. Each ring has at least one key to the locks of two different doors. No two rings carry keys for the same two doors.

There appear to be two different answers, depending on the interpretation of the wording of the problem. The following solution, from Timothy Chow, discusses one place where an interpretation is needed. Jonathan Hardis notes that another interpretation is needed: is each ring for exactly two doors or for at least two doors? Chow writes,

"I believe I have a solution to problem 2. It might not be the 'intended' solution, though; I remark on this at the end.

"In the worst case, four key rings are needed. Let the doors be A, B, C, D, E, and suppose door A has 12 locks, A1, A2, ..., A12, and each of the other doors has two locks (B1, B2, C1, C2, etc.). The rings could be

Ring 1: A1 A5 A9 B1
Ring 2: A2 A6 A10 C1
Ring 3: A3 A7 A11 D1
Ring 4: A4 A8 A12 E1
Ring 5: B2 C2 D2 E2

"As far as I can tell, this setup satisfies the constraints of the problem. If he picks the first three key rings then he is out of luck, so he has to pick four. Clearly, any four key rings will work.

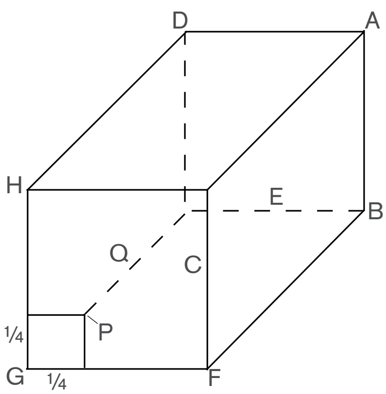
"Note that at first glance it appears that we could omit keys A5 through A12 entirely and still have a valid solution; however, in that scenario, the man could distinguish ring five from the rest because it has four keys instead of two keys, and so he could adopt the strategy of always picking the ring with four keys and one other ring. Introducing the locks A5 through A12 makes the rings indistinguishable from each other.

"To show that there is no configuration that requires the man to take all five rings, suppose toward a contradiction that such a configuration exists. Then there must exist a subset of four rings that omits at least one key for each door. It follows that the fifth (unchosen) ring has at least one key for every door. But the problem stipulates that every ring has at least one key to the locks of two different doors, so if you compare any of the first four rings with the fifth ring, there will be at least two doors having keys on both rings, which violates another stipulation of the problem.

"Now, as I remarked before, this might not be the intended solution. If every door has *exactly* two locks, then a nice graph-theoretic argument shows that three rings suffice: create a vertex for each ring and join two vertices with an edge if the rings have keys to the same door; then the conditions of the puzzle force the graph to be a five-cycle, and to make sure that you pick both ends of some edge, it suffices to pick any three vertices. This is my guess as to the intended problem. However, the problem as stated admits the solution I presented above."

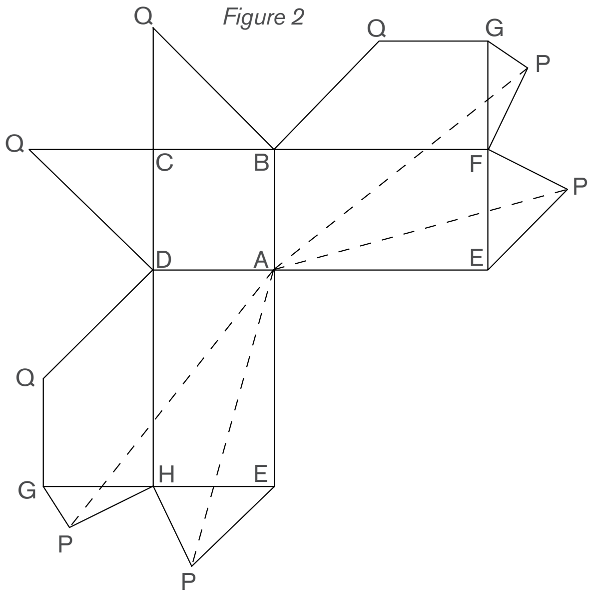
M/A 3. Consider a bug at a corner of a $1 \times 1 \times 2$ solid. Clearly, the farthest-away point is the diagonally opposite corner, if the bug can travel through the solid. But our bug is restricted to the surface of the solid (vertices, edges, and faces). What point is the farthest away? The extended problem is to find two points that are maximally far apart for the bug.

Figure 1



Charles Wampler claims to have fun with these problems. I am pleased that the results of his fun can be beautifully presented solutions such as the one below:

Figure 2



"For a bug traveling on the surface of a $1 \times 1 \times 2$ rectangular solid, the farthest point from a corner, say A, is not the diagonally opposite corner, say G (see Figure 1). It is a point, say P, on the opposing 1×1 face, ¼ the way along the diagonal from G. Interestingly, there are four shortest paths from A to P, all of length $|AP| = \sqrt{65/8}$. This is slightly larger than $|AG| = \sqrt{8}$ (for which there are two paths). Figure 2 (not to scale) shows an unfolding of the surface of the solid onto the plane such that the shortest path from A to any other point is a straight line. (By the way, point Q, the midpoint of edge CG, also has four shortest paths from A, all of length $\sqrt{5}$.)

"The two points of the surface that are maximally far apart are the centers of the 1×1 faces. These are three units apart."

OTHER RESPONDERS

Responses have also been received from R. Ackerberg, D. Aucamp, D. Boika, M. Boltin, D. Dechman, J. Feil, R. Giovanniello, T. Harriman, S. Harris, R. Haskell, K. Lebensold, Z. Moledina, J. Prussing, K. Rosato, and A. Sood.

PROPOSER'S SOLUTION TO SPEED PROBLEM

12.566.

These are the values where the area equals the perimeter.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.