I just returned from my 40th reunion at MIT, which I greatly enjoyed. My favorite parts were the Fenway Park tour and the Class of '67 brunch. Although the bagels were fine and Fenway was fascinating (even for a Yankee fan), the real pleasure was meeting several of my classmates, especially fellow denizens of Baker House. I plan to attend future reunions, especially my 45th and 50th. Yes, I am optimistic by nature.

Since this is the first issue of a new academic year, let me again review the ground rules. In each issue, I present three regular problems, the first of which is normally related to bridge (or chess or some game), and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e., four months) later, one submitted solution is printed for each; I also list other readers who responded. This issue, for example, contains solutions to regular problems posed in May/June.

I am writing this column in June and anticipate that the column containing the solutions will be due in October. Please try to send your solutions early to ensure that they arrive before my submission deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Respondents" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that remained unsolved when first presented.

PROBLEMS

S/O 1. We start with a bridge problem from Jorgen Harmse, who wants South to make seven hearts against an opening lead of the diamond ace.

North	♠ K J 10 8 6 2 ♥ 9 4 ♦ 9 7 2 ♣ 7 2	South	♠ ♥ A K Q J 10 7 6 3 ♦ ♣ A K J 6 3

S/O 2. A rather different type of "bridge" problem from Arthur Wasserman. There are four people who must cross a bridge at night; they have only one flashlight, and that is adequate to guide only two people at a time over the bridge. A can cross the bridge (in either direction) in one minute; B requires two minutes; C requires five minutes; and D takes ten minutes. How quickly can they manage to get all four across the bridge?

S/O 3. Howard Haber offers us another "three hat" problem, one that he heard in Geneva.

Three logical people, A, B, and C, are wearing hats with positive integers painted on them. Each person sees the other two numbers, but not her own. Each person knows that the numbers are positive integers and that one of them is the sum of the other two. They take turns (A, then B, then C, then A, etc.) in a contest to see who can be the first to determine her number. During round one, A, B, and C pass. In round two, A and B again pass, at which point C states that she now knows all three numbers and that their sum is 144. How did C figure this out?

SPEED DEPARTMENT

In honor of my class's winning the (age-adjusted) crew race at the reunion, I offer the following speed problem from Robert Ackerberg.

An MIT student, rowing upstream on the Charles River, passes under the Mass. Ave. bridge when his hat drops into the river without his noticing. After rowing for a bit, he realizes his loss and sees his hat floating downstream. He immediately turns to row toward the hat, and after 20 minutes, he reaches it and removes it from the river. At that moment, he is exactly one mile downstream of the bridge. What is the velocity of the water in the Charles?

SOLUTIONS

M/J 1. Howard Sard solved part one with the white king on h7, the white queen on h8, the black king on f7, a black pawn on e7, and a black rook on g4. If the king moves, Rh4 wins the queen. If the queen moves, either it can be captured or Rh4 is checkmate.

Richard Hess solved both parts, but I misstated the second part. It is White who must have more than 30 legal moves. The proposer's solution (to the correct problem) is Black: king on h8, rook on f6, bishop on g7, and pawn on a7. White: king on b2 and queen on d4. If the queen moves, the rook attacks (or captures) it, discovering a check on the king. If the king moves, the rook checks, discovering an attack on the queen.

M/J 2. Larry Bell clearly prefers bathing to cocaine (a wise choice). He writes,

"Call the nine-milliliter vial A and the ten-milliliter vial B. There are three procedures one can perform with the two vials, but the solution requires only two. Before the dissolution of the pill, an initializing step can produce any integer number of milliliters of water. The other procedure could be called 'splish splash': if A contains nine milliliters of fluid plus x milligrams of solute, and B contains nine milliliters of fluid plus y milligrams of solute, one can pour one milliliter of solution from A to B and then back again from B to A. This results in final solute amounts of $(9x + y)/10$ in A and $(x + 9y)/10$ in B.

"To localize exactly 41 percent of the solute in one of the vials, proceed as follows: Initialize to produce four milliliters of pure water in A. Dissolve the pill in vial B in ten milliliters of water. Fill A from B, transferring five milliliters and 50 percent of the solute from B to A. Discard contents of B, then transfer all nine milliliters and 50 percent of solute from A to B. Fill A with nine milliliters water. Splish splash. Now B contains 45 percent of the solute. Splish splash again. Now B contains 41 percent of the solute."

Fluid in A	Fluid in B	Med. in A	Med. in B	Pour out	Fluid in A	Fluid in B	Med. in A	Med. in B	Pour out
0	10	0	0		9	4	0	0	
9	1	0	0		0	4	0	0	9
0	1	0	0	9	4	0	0	0	
1	0	0	0		4	10	0	100	
1	10	0	0		9	5	50	50	
9	2	0	0		9	0	50	0	5
0	2	0	0	9	0	9	0	50	
2	0	0	0		9	9	0	50	
2	10	0	0		8	10	0	50	
9	3	0	0		9	9	5	45	
0	3	0	0	9	8	10	$4\frac{4}{9}$	$45\frac{5}{9}$	
3	0	0	0		9	9	9	41	
3	10	0	0						

M/J 3. Steve Goldstein chose to keep Doyle's seven instead of the solution. He notes that the decimal representations of the sevenths 1/7, 2/7, …, 6/7 all consist of infinite repetitions of the string 142857 (starting at different digits: e.g., 1/7 starts with 1, 3/7 starts with 4). Goldstein wonders if in base 10 there are other denominators and strings that share this property with 7 and 142857. What about other bases?

The following solution is from Timothy Chow: "The 142857 puzzle is a well-studied mathematical problem in slight disguise. Let b be the base, and let p be the number whose reciprocal is of interest. Since the numbers $1/p$, $2/p$, …, $(p-1)/p$ are all distinct and are supposed to correspond to different cyclic permutations of the same string, that string must be at least $p - 1$ digits long. From the division algorithm it is easy to see that this means that the numbers b, b^2, b^3, …, b^{p-1} must all be distinct modulo p. It follows that p must be prime and that, in number-theoretic jargon, b must be a primitive root of $(Z/pZ)^*$, the multiplicative group of the integers mod p. Conversely, if b is a primitive root mod p, then it yields a solution to the original puzzle.

"A basic fact of number theory is that $(Z/pZ)^*$ is cyclic, which means that for any prime p there must exist a primitive root b. If b works, then so does anything congruent to b mod p. Furthermore, there are exactly $\phi(p-1)$ distinct possibilities for b mod p, where ϕ is the Euler phi function ($\phi(x)$ is the number of positive integers less than x that are relatively prime to x).

"So for fixed p, there are infinitely many b that work, and we can compute them. The converse assertion, that for any fixed b (except for perfect squares, which can't work) there are infinitely many p that work, is a notorious open problem known as Artin's conjecture. (See en.wikipedia.org/wiki/Artin's_conjecture_on_primitive_roots.)

"It turns out that it suffices to check prime numbers b. The best result so far is that of Heath-Brown, who showed that Artin's conjecture is true with at most two exceptions. But we don't know what the exceptions are, so in particular we don't know whether there are infinitely many primes p that work for $b = 10$ (or for any particular value of b, for that matter)."

Allan Trojan gave a similar analysis and notes that, for example, 7, 17, 19, and 23 work for base 10, and 11, 13, and 17 work for base 6. Richard Hess notes that 2 works for any even base, and 5 works for bases 2, 3, 7, and 8.

BETTER LATE THAN NEVER

2006 J/A 2. Didir Bizzarri gives a construction method for the tangramoid.

OTHER RESPONDERS

Responses have also been received from R. Ackerberg, R. Giovanniello, S. Gorovitz, J. Kenton, J. Martinez, Z. Moledina, and K. Rosato.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Take a frame of reference fixed to the hat, so the problem can be solved in still water. At first, the hat and boat are together; they separate and then come back together again. If it took 20 minutes for the boat to reach the hat when the rower turned around, then the time elapsed when they were separating is also 20 minutes. So in 40 minutes, the hat moved one mile downstream, and the water velocity is $1/(40/60) = 1.5$ miles per hour.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.