It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a large supply of regular problems, less that a year's worth of speed problems, and an 18-month backlog of bridge (chess, etc.) problems.

Auran reports that he has sent tear-out pages of "Puzzle Corner" to the National Cryptologic Museum in Ft. Meade, MD. I worry that this will provide the young cryptologists in my (NYU Computer Science) department further evidence that I have become a museum piece.

PROBLEMS

M/J 1. We begin with a related pair of problems proposed by Ira Rosenholtz and Andy Liu. For the first problem, you are to construct an endgame where White has just a king and queen; Black has just a king, a rook, and a pawn on its initial square; it is White's move; and Black wins. For the second problem, Black has an additional bishop, and we are told that he has more than 30 legal moves.

M/J 2. Richard Hess has a variation on Sir Arthur Conan Doyle's seven percent solution: given a 10-milliliter vial, a 9-milliliter vial, a water supply, and a single water-soluble tablet of medicine, how can one measure out a dose (dissolved in water) of exactly 41 percent of the tablet?

M/J 3. Steve Goldstein chose to keep Doyle's seven instead of the solution. He notes that the decimal representations of the sevenths 1/7, 2/7, ..., 6/7 all consist of infinite repetitions of the string 142857 (starting at different digits: e.g., 1/7 starts with 1, 3/7 starts with 4). Goldstein wonders if in base 10 there are other denominators and strings that share this property with 7 and 142857. What about other bases?

SPEED DEPARTMENT

Ermanno Signorelli apparently has a fascination with grasshoppers and jade knives. He writes, "Grasshopper is asked by his master to balance the flat-bladed jade knife horizontally, at its hilt, and he is unable to do so. The master asks him to observe the milk-white onyx bowl on the monastery altar. He is then told to place the flat blade over the hollow of the bowl, with the hilt on the very edge of the bowl. Grasshopper is astounded to find that the knife remains stationarily horizontal. What is the characteristic of the bowl that causes this to happen?"

SOLUTIONS

J/F 1. Richard Hess supplied the only solution! He notes that Friedman and Jungreis's "How Many Bridge Auctions" (www.stetson.edu/~efriedma/papers/bridge.pdf) shows that the number is $4(2^{35})/3 \approx 1.28746 \cdot 10^{47}$.

(I guess we will trust this answer even though Jungreis was a student at Harvard. I couldn't resist, sorry.—Ed.)

Hess then notes that an upper bound on the number of ways to play a hand where each player is dealt 13 cards is for each player to have 13! choices, and there are 4 possible choices for who plays the very first card. Since $4(13)! \approx 6.014575 \cdot 10^{39}$, there are more possible auctions than ways to play a hand.

J/F 2. This was a *very* popular problem, and I must say that the answer surprised me. The following solution is from Walter Sun.

"Surprisingly, the first pirate can walk away with both his life and 97 of the 100 gold coins if he makes the correct proposal. To reach this conclusion, the first pirate should work backwards from the endgame to determine optimal behavior. With two pirates left, the last pirate is guaranteed 20 coins if he refuses the fourth pirate's proposal. Thus, to avoid execution, the fourth pirate must offer the fifth pirate at least 21 coins (and no more, since he knows that the fifth pirate is maximizing his outcome).

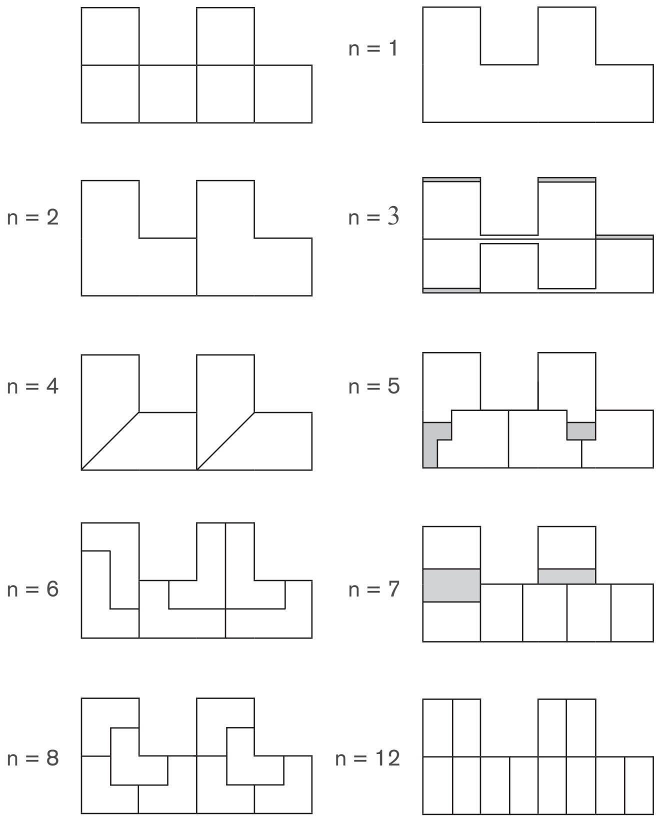
"With this information, the third pirate knows that he can get a majority simply by getting either of the two other pirates to agree to his proposal. Since the fourth pirate would keep only 19 coins if he had to make a proposal, he is satisfied with an offer of 20 coins. With this, the third pirate knows that he will win the vote regardless of what the fifth pirate decides, so he can offer the fifth pirate 0, thus leaving himself with 40 coins.

"The second pirate needs two other pirates to be happy to get a majority. Again, he prefers to satisfy the fourth and fifth pirates because an offer of 21 and 1 coins, respectively, would result in both saying yes (as opposed to offering 41 to the third pirate). The third pirate gets nothing, and the second pirate gets to keep 58 coins.

"Now, the first pirate knows that he just needs the vote of two of the other four pirates. So he offers the third pirate one coin and the fifth pirate two coins, both better outcomes than would result if the second pirate gets a chance to make his proposal. The second and fourth pirates are offered nothing, since their votes don't matter. As a result, the first pirate keeps the balance of the coins, which is 97."

J/F 3. Avi Ornstein found solutions for $n = 1, 2, ..., 8, 12$, most of which are pictured below. For $n = 3$, we show instead the solution from the proposers, which has the property that the uncovered area can be made arbitrarily small. The percentage of the area covered is

n	%	n	%	n	%
1	100	4	100	7	87.5
2	100	5	93.75	8	100
3	$1-\varepsilon$	6	100	12	100



Note that the original diagram, printed as the upper left drawing, is actually an alternate solution for $n = 6$.

BETTER LATE THAN NEVER

2005 Sep 3. Tom Terwilliger has shown both analytically and via simulation that as the number of pills in the bottle increases, the probability goes to zero that the penultimate tablet removed is a half tablet.

Y2006. Marlon and Levi Weiss noticed that 30 should be 60/2+0.

2006 S/O 1. Frank Rubin's Web page, www.contestcen .com/knight.htm, shows knights covering chessboards of many different sizes. For example, 350 knights can cover a 50-by-50 chessboard.

S/O 3. As Don Aucamp, Jonathan Hardis, Peter Lobban, and J. Walkinshaw politely pointed out, I blew this one. Sorry. The problem stated that the balance can hold at most two coins on each side, but the printed solution used three coins per side. Fortunately, I did not discard the other solutions received and now print the one from Aaron Ucko.

"Yes, it is possible. If we label the balls with letters (from A to I), then weighing A + B against C + D and A + E against B + F divides the 72 possible outcomes into nine groups of eight, each of which is possible to resolve in two further weighings. Specifically:

"If A + B = C + D and A + E = B + F, then we can find both altered balls in either {C, D} or {G, H, I}, at which point comparing G to H and C to I will reveal the details.

"If A + B = C + D and A + E > B + F, then there are four possible explanations: A is heavy and B is light; E is heavy and F is light; E is heavy and either G, H, or I is light; and F is light and either G, H, or I is heavy. Comparing E + F to G + H will mostly distinguish these cases (lumping the first two together); comparing either A to E (if E + F = G + H) or G to H (otherwise) will reveal the remaining details. The case A + B = C + D and A + E < B + F is analogous.

"If A + B > C + D and A + E = B + F, then there are also four possible explanations: A is heavy and E is light; B is heavy and F is light; C is light and G, H, or I is heavy; and D is light and G, H, or I is heavy. Comparing C to D will mostly distinguish these cases (lumping the first two together here as well); comparing either A to B (if C = D) or G to H (otherwise) will reveal the remaining details. The case A + B < C + D and A + E = B + F is analogous.

"If A + B > C + D and A + E > B + F, then either A is heavy and C, D, F, G, H, or I is light or E is heavy and C or D is light. To break these cases down, one can compare C + F to D + G and follow up by comparing H to I if they're equal, A + D to B + C if C + F > D + G, and A + C to B + D if C + F < D + G. The remaining three cases are analogous."

2007 J/F SD. David Brahm and Michael Pappas noticed that 128 5/7 should be 128 4/7.

OTHER RESPONDERS

Responses have also been received from J. Chonoles, G. Coram, T. Curtis, C. Danielian, J. Feil, M. Foringer, R. Giovanniello, A. Goodisman, J. Grossman, R. Hakala, W. Hughes, D. Katz, J. Kesselman, T. Kim, J. Kosch, A. Laves, C. Lee, J. Martinez, Z. Moledina, M. Myers, L. Neo, C. Perry, J. Prussing, K. Rosato, J. Simon, G. Smith, E. Specht, A. Taylor, M. Viswanathan, F. Webb, D. Weber, P. Weiss, C. Whittaker, D. Zalink.

PROPOSER'S SOLUTION TO SPEED PROBLEM

The bowl is filled to the brim with milk that is the same color as the bowl. Surface tension causes the flat blade to adhere to the milk. Ermanno reports that he has a 24-centimeter knife that does so balance.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.