This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 7) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2006 yearly problem is in the "Solutions" section.

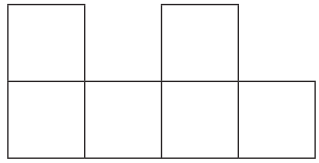
Problems

Y2007. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 7 exactly once each, the operators +, –, × (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators, and among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 7 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator.

J/F 1. Guy Steele poses what might be called a "combinatorial bridge problem." Once a bridge hand has been dealt, are there more ways to bid it or to play it? That is, compare the total number of legal auctions with the total number of legal ways to play out the cards. No insufficient bids, revokes, bids or plays out of turn, etc.

J/F 2. Howard Haber's visit to England produced this appropriately seafaring problem. Five pirates have to divide 100 gold coins among them. There is a pecking order among the pirates, so pirate number one makes the first proposal for how the coins should be divided. The pirates take a vote. If a majority of the pirates vote to accept the proposal, the deal is done. All votes get counted, including that of the lead pirate, who made the proposal. If the proposal fails to win a majority (a tie is not good enough), then the lead pirate is killed, and 20 gold coins are paid to the executioner (who is not one of the five pirates). The next pirate in the pecking order takes over and makes a new proposal on how to divide up the remaining 80 gold coins. The process continues until either the vote yields a majority, or there is one pirate left. Assume that each pirate votes in a way that maximizes his final share (if a vote does not affect the outcome, he votes "no"—after all, they're pirates!). What proposal should the first pirate make to maximize his final outcome?

J/F 3. Our final problem is from the Robert Hess and Robert Wainwright "Modest Hexominoes" series. You are to design a connected tile, *n* of which cover the maximum area of the hexomino below. The *n* tiles are identical in size and shape and may be turned over so that some are mirror images of others. They must overlap neither each other nor the border of the hexomino.



Speed Department

Avi Ornstein wants to know the next four numbers in the series beginning 60, 90, 108, 120.

Solutions

Y2006. As with Y2007, offered in the "Problems" section above, you are asked to form integers from 1 to 100 using the digits 2, 0, 0, and 6 exactly once each, the operators +, –, ×, /, and exponentiation. We prefer solutions using the minimum number of operators and the digits in the order 2, 0, 0, 6.

The following solution is from Avi Ornstein:

1	206^0	*12	(2+0+0)*6	40	60–20
2	2+0*60	*14	20–0–6	58	60–2–0
3	60/20	19	20–6^0	59	60–2^0
*4	-2+0+0+6	*20	20+0^6	60	60+0*2
*5	-20^0+6	21	20+6^0	61	60+2^0
*6	20^0*6	*26	20+0+6	62	62+0+0
*7	20^0+6	30	60/20	*64	(2+0)^(0+6)
*8	2+0+0+6	36	6^2+0+0	80	20+60

S/O 1. Fred Tydeman seeks the minimum number of knights that can be placed on an otherwise empty chessboard so that each of the 64 squares is either occupied or under attack.

Ed Sheldon has sent us a complete solution containing a proof of minimality and a rather detailed diagram showing the knights' placement and coverage.

"The minimum number of knights is 12. First, it can be easily shown that the number must be at least 12. Consider the upper left-hand corner, a7, a8, b7, and b8. The squares a7 and b8 can be attacked by one knight at c6, but a8 and b7 cannot be attacked simultaneously. Thus at least three knights are needed for this corner. The lowest rank from which a knight can attack the seventh rank is the fifth rank, but a knight on the fifth rank can attack only as low as the third rank and thus cannot attack the first and second ranks of the lower left corner, etc. Thus each two-by-two corner requires a minimum of three knights, which, multiplied by four corners, gives 12 as the lowest number of knights that can attack the corners and thus the minimum that can cover the board. A solution with 12 knights was found with a little trial and error and the assumption of rotational symmetry:

"The knights are placed at the capital letters, and each lowercase letter indicates the knight or one of the knights that can attack that square. The colored squares show the pattern that was rotated."

e	a	e	a	c	k	c	f
a	i	k	c	a	F	k	c
i	E	A	f	K	C	j	f
a	d	I	c	a	g	b	c
d	a	e	a	c	J	c	b
h	i	D	L	h	B	G	j
d	l	H	b	d	l	j	b
h	d	l	d	b	g	b	g

Eric Nelson-Melby found, in addition, that for n from 1 to 7, the number of knights needed is 1, 4, 4, 4, 5, 8, and 10.

S/O 2. My old Baker House colleague John Rudy plays racquetball with his son. In this game, if the server wins the rally, he gets a point and serves again. If the server loses the rally, no points are awarded, and the opponent serves next. Assume that whoever is serving wins the rally with probability p; the score is server 13, opponent 14; and 15 points wins the game. For what values of p is the server more likely to win than the opponent?

I enjoyed reading the solutions to this problem. The following response from George Blondin, like several others, avoids infinite series.

"Let x, y, and z be the odds of the server winning the game when the score is 14-14, 14-13, and 13-14, respectively. Then

$$x = p + (1 - p)(1 - x)\text{; so } x = 1/(2 - p)$$

$$y = p + (1 - p)(1 - z) = 1 - z + pz$$

$$z = px + (1 - p)(1 - y) = p/(2 - p) + 1 - p - y - py$$
$$= p/(2 - p) + z - 2pz + p^2 z$$

Multiplying the last equation by $2 - p$ and simplifying yields $4z = 1 + 4pz - p^2 z$.

"Substituting $z = .5$ and solving the quadratic gives the minimum probability $p = 2 - \sqrt{2}$."

S/O 3. Victor Luchangco had nine coins of equal weight, but someone removed material from one coin and added it to another. Victor has a balance scale that can hold at most two coins on each side and wishes to determine *both* the lighter and heavier coin using only four weighings. Can he do it?

As Robert Hess notes, the solution in Tim Sole's *The Ticket to Heaven and Other Superior Puzzles* has the added property of independent weighings. That is, later weighings do not depend on the results of previous weighings. Label the coins A, B, ..., I. The first weighing is ABC vs. DEF; the second weighing is AEG vs. BDH; the third weighing is AD vs. BE; the fourth weighing is CF vs. GH.

Each square in the table below shows the results of the four weighings when material has been removed from the coin indicated by the row label and added to the coin indicated by the column label. The symbols +, - , and 0 indicate that the left scale pan is heavier, lighter, or balanced, respectively.

Heavy coins

	A	B	C	D	E	F	G	H	I
A	0000	0--0	0--+	--00	-0-0	---+	-0--	----	---0
B	0++0	0000	0+++	-0+0	-+00	-+++	-++-	-0+-	-++0
C	0++-	0---	0000	--+-	-+--	-000	-+0-	--0-	-00-
D	++00	+0-0	++-+	0000	0+-0	0+-+	++--	+0--	++-0
E	+0+0	+-00	+-++	0-+0	0000	0-++	+0+-	+-+-	+-+0
F	+++-	+---	+000	0-+-	0+--	0000	++0-	+-0-	+00-
G	+0++	+--+	+-0+	--++	-0-+	--0+	0000	0-00	0-0+
H	++++	+0-+	++0+	-0++	-+-+	-+0+	0+00	0000	0+0+
I	+++0	+--0	+00+	--+0	-+-0	-00+	0+0-	0-0-	0000

Other Responders

Responses have also been received from D. Aucamp, G. Cheng, C. Dale, W. Eddleman, D. Emberson, R. Giovanniello, J. Hardis, O. Helbok, H. Ingraham, J. Karnofsky, J. Kenton, P. Kramer, U. Mobin, K. Rosato, M. Rothkopf, B. Rothleder, M. Seidel, A. Shuchat, E. Signorelli, S. Silberberg, T. Stellmach, A. Taylor, A. Ucko, and S. Ulens.

Better Late than Never

S/O 2. The solution published was from Bob Ackerberg, whose name was inadvertently omitted. I apologize for the error.

Proposer's Solution to Speed Problem

$128\frac{5}{7}$, 135, 140, 144. These are the measures of each angle in an equilateral triangle, square, regular pentagon, etc.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.