It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a comfortable supply of regular, bridge/chess, and speed problems.

Problems

M/J 1. Larry Kells found an "amazing deal" by Thomas Andrews where all four players can make 3NT as declarer, against best defense. Can you find one?

M/J 2. Warren Smith, a former colleague of mine at the NEC Research Institute, advocates "range voting" for elections. The following problem is from his Center for Range Voting website, math.temple.edu/~wds/crv. Three people enter a room, and a red or blue mark is made on each person's forehead. The color of each mark is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the others' marks but not his own. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other marks, they must (simultaneously) guess the color of their own marks or pass. The group shares a hypothetical million-dollar prize if at least one player guesses correctly and none guess incorrectly. The problem is to find a strategy for the group maximizing its chances of winning the prize. For example, one obvious strategy for the players would be for one player always to guess "red" while the others pass. That would give the group a 50 percent chance of winning the prize. Can the group do better?

M/J 3. Our final problem is the originally intended version of 2005 Jul 2 as submitted by Avi Ornstein (I inadvertently changed it).

Given that a right triangle with legs of lengths A and B is circumscribed around a circle of radius one, express the length of B in terms of A. In addition, what is the smallest possible area of the triangle?

Speed Department

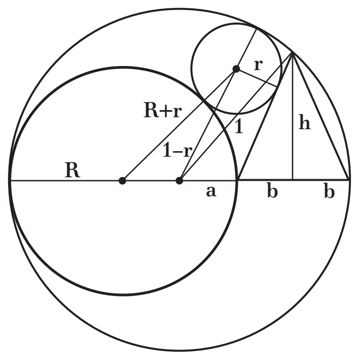
SD. Richard Hess is playing a set of tennis. In the last eight points, his opponent has served seven aces, and he has served one ace. What is the score?

Solutions

DEC/JAN 1. In solving Jorgen Harmse's chess problem, Mark Moss found the key first move of promoting the pawn to a knight. Black has five replies, the best of which is king to f7. Then we have QxR, QxQ (else white continues QxQ), NxQ. Now, White can use the three knights to checkmate the black king (a known ending). If Black first moves the king to a square other than f7, White can mate in at most four additional moves. For example, 1. ... Kh5; 2. Qe5+ Kh4; 3. Qg3+ Kh5; 4. Qg4+ Kh6; 5. Qg6 mate.

DEC/JAN 2. Joel Karnofsky writes, "As you will see, I find that there are two solutions to Phil Cassady's plane geometry problem that seem to be consistent with the problem specification, for only one of which the desired result is true. Oh the perils of relying on pictures." Karnofsky's full solution can be found on the "Puzzle Corner" Web page at cs.nyu.edu/~gottlieb/tr; an edited version, still including his clarification of the problem statement, follows.

"Given a circle of radius 1, place an isosceles triangle with base $2b$ and height h so that its base overlaps the right end of a diameter of the circle and its top is on the circle. Place a circle of radius R so that one of its diameters coincides with the remainder of the first circle's diameter. Add a third circle of radius r that is tangent to the two circles and the interior side of the triangle. (The puzzle says, "add a third circle inscribed so that it touches the other two circles and the triangle.") The puzzle is to show that the center of the third circle is directly above the interior corner of the triangle. We will actually show that there are two solutions consistent with the specification, only one of which has the desired property.



"Refer to the picture at left. Our approach is essentially algebraic. Let (x, y) be the coördinates of the center of the third circle, relative to an origin at the interior corner of the triangle. Let a be the distance from the center of the main circle to this corner. Simple relations on the main diameter give $2R + 2b = 2$ and $a + 2b = 1$. Since a radius is perpendicular to a tangent line, the line between the center of the second and third circles gives $(r + R)^2 = (x + R)^2 + y^2$. The line between the center of the main and third circles gives $(1 - r)^2 = (x + a)^2 + y^2$. The line between the center of the main circle and the top of the triangle gives $1 = (a + b)^2 + h^2$.

"The line containing the interior side of the triangle consists of all points $(0, 0) + s(b, h)$ for real numbers s. The line from the center of the third circle perpendicular to this line consists of all points $(x, y) + t(-h, b)$ for real numbers t. Equating these two gives the intersection point at $t = (hx - by)/(b^2 + h^2)$. The length of the segment from the center of the third circle to this point gives $r^2 = (hx - by)^2/(b^2 + h^2)$.

"Solving all these equations, leaving only b as an independent variable, gives four solutions consistent with the triangle being on the right and pointing up.

"One of these is

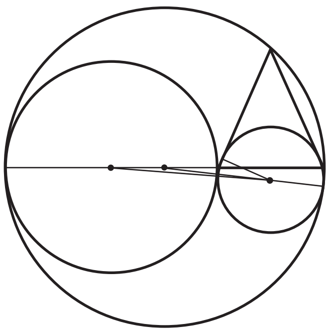
$$x = 0,\ r = \frac{2\,(-1+b)b}{-2+b},\ y = \frac{\sqrt{2r}}{\sqrt{b}}$$

which has the desired property that $x = 0$ and matches the picture above with $b = ⅓$.

"A second solution is

$$x = \frac{4\sqrt{2}(-2+b)\sqrt{-(-2+b)^3(-1+b)^2b^4 + 4(-1+b)b^2(4-10b+5b^2)}}{b(-4+2b+b^2)^2},$$

with r a similar-looking fraction and y a mess. For this solution, except at the extremes, $x \neq 0$. The corresponding picture for $b = ⅓$ is



"In the two remaining solutions, the third circle, located in the lower horn between the first two circles, is tangent to the side of the triangle only when the side is extended into the lower half of the main circle, which presumably violates the intent of the problem.

"My statement of the problem potentially allows another type of solution, where the third circle is simultaneously tangent to the first two circles at the left end of the main diameter. However, in this case, the third circle is again tangent only to the extension of the triangle's side.

"In all the solutions, x, r, and y are constructible numbers, which implicitly gives a ruler-and-compass construction for the third circle."

DEC/JAN 3. I was at first confused why several correct-looking solutions to Ludwig Chincarini's Parkinson's disease problem gave the same "wrong" answer; then I noticed that the font makes the "3" (three) in .3 percent look like a "5" (five). Please address any complaints to the Berthold type foundry in Berlin, which created the font.

Several readers simply plugged the values given into Bayes's theorem, and out popped the correct answer. For the benefit of readers not familiar with this theorem, I have chosen Robert Sherrick's solution, which essentially contains the derivation of the theorem. Sherrick gave his solution with a fixed-size population of 100,000, but the argument is the same if one replaces 100,000 with N. He writes,

"This is a common problem encountered in medicine-screening asymptomatic populations for the presence (or in this case, the probability of developing) a disease. The simplest way to solve it is to imagine screening a given population and calculating the results. Say we were to screen 100,000 patients with this test (I have used this number because it makes all the answers integers):

"Of the 100,000 patients, 300 will develop Parkinson's disease, and 99,700 will not. Of the 99,700 who will not develop Parkinson's disease, 92,721 will test negative, and 6,979 (or 7 percent) will test positive. Of the 300 who will develop Parkinson's disease, 279 will test positive, and 21 (or 7 percent) will test negative.

"The probability of developing the disease given that you have a positive test is 279/(279 + 6,979) = 3.844 percent. This is known as the positive predictive value. The probability of developing the disease if you have a negative test is 21/(21 + 92,721) = .02264 percent. This is known as the negative predictive value."

Better Late than Never

2005 JUL 3. Jim Francis explains that since "oid" means "like" or "similar to," I would have been successful had I searched for "tangram," and indeed Google gives many hits. Francis adds that "you could even (gasp!) look it up in a dictionary!!" R. V. Baum actually did this and found "a Chinese puzzle consisting of a square cut into five triangles, a square, and a rhomboid, which can be combined so as to form a variety of other figures." Robert Talambiras asserts that the "oid" suffix "is used by relative youth to be attached to almost anything," which may explain why someone like me, who is no longer even remotely related to youth, was fooled. The pictures included in the problem remind Rocco Giovanniello of those produced by a kaleidoscope.

2005 MAY SD. Jared Black, Krzysztof Kopanski, and John Prussing offer nonspeed solutions without any assumptions.

Other Responders

Responses have also been received from R. Ackerberg, M. Baum, F. Baum, F. Carbin, P. Cassady, F. Crespo, D. Diamond, S. Feldman, E. Field, D. Frankel, L. Freilich, M. Garrison, R. Giovanniello, S. Haber, G. Hambleton, R. Hess, H. Hodara, H. Ingraham, S. Kayton, J. Kenton, P. Kramer, N. Lang, R. Lenoil, A. Lipsky, J. Pinson, E. Passow, M. Rothkopf, C. Russ, E. Sard, M. Segal, D. Seldin, A. Shuchat, J. Stark, C. Tavares, M. Thompson, D. Watson, M. Weiss, S. Wisotsky, and P. Worfolk.

Editor's Solution to Speed Problem

From love-40, the first five aces brought the set to 6-6. Richard served first in the tiebreak, so the score is 6-6 (3-1).

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York NY 10003, or to gottlieb@nyu.edu.