PUZZLE CORNER

IT HAS BEEN a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a large supply of regular problems and Bridge/Chess problems and an adequate, but smaller, supply of speed problems.

Problems

MAY 1. An unusual Chess problem from Jorgen Hamse. Stalemate occurs when the side to move has no legal move, but is not in check. In (some versions of) speed Chess, moving the King into check is legal; the appropriate reply being to capture the King and claim the game. Can you find a position where neither side is in check and neither side has a legal move, even considering moving into check legal? The position must be reachable; that is, there must be two sequence of legal (perhaps bizarre) moves leading to this position, one with White to play the other with Black to play.

MAY 2. In 1996, we asked how to place lamp posts to illuminate the equator of a planet. Now Andrew Russell wants you to place the minimum number of lamp posts needed to illuminate the entire (spherical) planet. Oh yes, you are also to arrange that the total length of all the posts is minimal (among solutions with the minimal number of posts).

MAY 3. Chuck Haspel has, among other problems, one involving the hands of a clock. He writes.

My wife and I had an argument a few months ago about which hand on the clock was the "big hand" (we have been married a long time.) I said the hour hand was the big hand and was supported by a tiny minority of the sources we checked; one Internet document, one of our children and one friend. All the rest supported her, and so did the fact that on many clocks the hands are the same width and the minute hand is longer.

This mini-contretemps will never be settled, but it does suggest the following problem.

Suppose you have a clock that can be read to infinite accuracy, but on which the hands are identical. Call a position of the hands "unambiguous" if there is no doubt about the time. 12:00 is unambiguous because the hands coincide, and 6:00 is unambiguous because if the hand on 6 were interpreted as the minute hand, the other hand would have to be halfway between two numbers. Are there any ambiguous positions, i.e., are there any legal positions of the hands which can be interpreted as two different times?

Speed Department

Albert Mullin wants to know what are the minimum and maximum number of "Friday the Thirteenths" that can occur in any year?

Solutions

DEC 1. Apparently the maxim that "crime doesn't pay" does not apply to Bridge, at least according to Larry Kells who wants to knowwhat is the largest number of tricks that can be gained by one revoke? Assume a two-trick penalty with no further adjustment.

Victor Barocas found this hand where cheating gains 10 tricks.

North
♠ J 2
♥ 2
♦ 6
♣ T 9 8 7 6 5 4 3 2

West
♠ A T 9 8 7 6 5 4 3
♥
♦ 5 4 3 2
♣

East
♠ Q
♥ A K Q J T 9 8 7 6 5 4 3
♦
♣

South
♠ K
♥
♦ A K Q J T 9 8 7
♣ A K Q J

The contract is in no trump (the size of the bid is irrelevant), and South is declaring.

If West leads AS, dummy's JS takes any spade continuation, and South claims by leading a diamond or club from the board. Obviously, a diamond switch by West is futile.

If West leads a small spade, South claims.

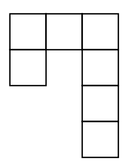
If West leads a diamond, South runs the minor-suit winners and concedes a spade at the end.

Thus, any legal line produces at most 1 trick for E-W.

If, however, West leads AS and East pitches a heart, then West can continue a spade to East's Q, and East claims the remaining tricks, producing 13 tricks for E-W. Assuming only a 2-trick penalty (which, of course, is not what would happen with any competent director), it's a net +10 for E-W.

I believe that this is the maximum. The two-trick penalty means that the absolute maximum possible would be +11 (from 0 to 13), but I believe that to be impossible. In a suit contract, whoever has the ace of trump must win it no matter what, so there can't be a 13-trick swing. In a no-trump contract, the revoke can't affect the outcome of the trick on which it occurs, so only 12 tricks can change.

DEC 2. Nob Yashigahara was asked by Junk Kato to cut the letter 7 below (made from 7 squares) into several pieces and re-arrange them into a perfect square. Can you do it?



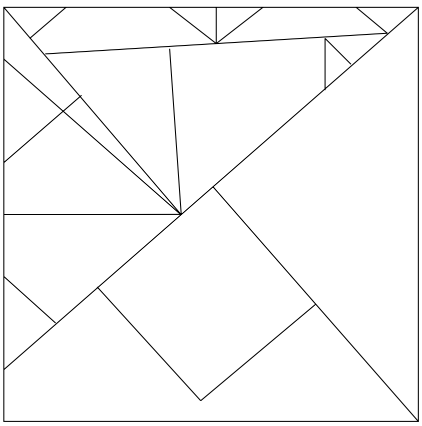
This appears to me to be quite a difficult problem. A few readers found a cute "solution": Cut out the center top box of the number seven and move it one row down. This gives the number four a "perfect square". Other readers cut the seven into pieces and reassembled it to form a geometrical square, but with a hole inside.

However, I believe the intent was to cut the seven into pieces, which are then rearranged to form a geometrical square without a hole Don Dechman tells us how to do so assuming you, like Pythagoras and Euclid, know how to convert squares on the legs of a right triangle into a square on the hypotenuse. (Dechman sent us the web reference http://kr.cs.ait.ac.th/~radok/math/mat2/chap62.htm, in case you need a refresher.) Given the above result, Dechman cuts the figure seven into its seven component squares and proceeds as follows

First, take two 1 × 1 squares, using a right triangle with unit sides, to construct a square with $\sqrt{2}$ sides. Then apply the constructed square and another 1 × 1 square, to a right triangle with one unit side and one $\sqrt{2}$ side, to construct a square with $\sqrt{3}$ sides. Finally apply this square and a 2 × 2 square (constructed from the remaining four unit squares) to a right triangle with one side two units long and the other side $\sqrt{3}$ units long, to construct the desired square that has $\sqrt{7}$ sides.

As an optimization, he note that the 2 × 2 square can be made from the 3 square "el" shaped upper right of the seven plus one 1 × 1 square. This optimization reduces the amount of cutting needed.

The diagram below shows the resulting $\sqrt{7}$ x $\sqrt{7}$ square composed of 19 pieces.



Eugene Sard sent a beautiful set of drawings that solves the problem without appealing to Pythagras and Euclid. I am posting Sard's solution on my "Puzzle Corner" web site http://cs.nyu.edu/~gottlieb/tr

DEC 3. Norman Spenser would like to know the radius of the inscribed circle for a triangle with side lengths a, b, and c. He would also like to know the radius of the circumscribed circle, but fears that might be harder.

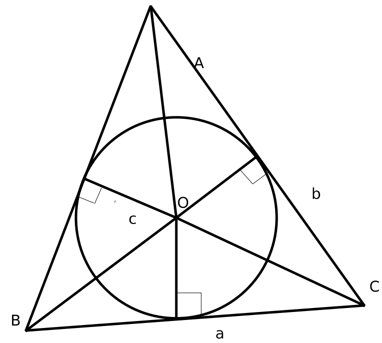
Geoffrey Kandall made this look easier than I thought it would be, given Heron's formula for the area of a triangle in terms of the semiperimeter. He writes.

From a, b, and c we first calculate the semiperimeter *s* and the area *K* via

$$s=½(a+b+c)$$

and

$$K=\sqrt{s(s-a)(s-b)(s-c)} \text{ (Heron's formula).}$$



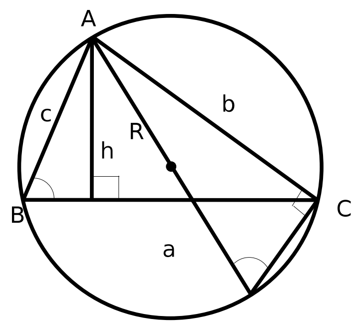
To determine the inscribed radius *r*, we use [XYZ] for the area of triangle XYZ and see from the diagram above that

$$[ABC]=[AOB]+[BOC]+[AOC]$$

or

$$K=½rc+½ra+½rb$$

Hence $r=K/s$.



For the circumscribed radius *R* we note that triangle ABC has at least one interior altitude, say the one from A to BC. From the second diagram we see that

$$\sin B=h/c=b/2R$$

Hence

$$R=bc/2h=abc/2ah=abc/4K$$

Other Responders

Responses have also been received from J. Alperin, Auran, P. Balbus, B. Brasher, G. Downie, L. Gingold, R. Giovanniello, T. Harriman, R. Haskell, X. Hutson, D. Kennedy, J. Kesselman, G. Knudsen, R. Merrifield, A. Ornstein, F. Powsner, C. Reimers, E. Sard, H. Sard, J. Serrao, E. Signorelli. B. Simon, N. Spencer, T. Terwilliger, J. Wouk, and J. Zachary.

Proposer's Solution to Speed Problem

1 and 3.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York NY 10003, or to gottlieb@nyu.edu.