THIS BEING THE first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0, 5) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2004 yearly problem is in the "Solutions" section.

Problems

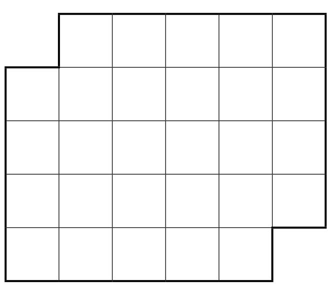
Y2005. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 5 exactly once each and the operators +, –, x (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 5 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign *does* count as an operator.

I realize that years that contain two zeros yield comparably few solutions to our yearly problem; we can all look forward to 2011, with only one zero, and 2013 with four distinct digits.

Since I am often asked why I do not permit 0^0, let me explain my reasoning. It is true that $x^x = 1$ for all positive x, so one might define 0^0 to the one by "continuity," as several readers have suggested. However, $0^x = 0$ for all positive x, which would suggest that 0^0 be defined as zero—also by "continuity." This argument shows there is no single value for 0^0 that continuously extends the values of x^y, for all positive x and y. Thus, I (and many others) consider 0^0 to be undefined.

MAR 1. Our first regular problem is a bridge offering from Larry Kells, who has been thinking about guaranteed contracts, i.e., single hands strong enough to guarantee that a certain contract is makable with best play for any distribution of the remaining cards. He specifically asks if there is any guaranteed suit contract in which the visible hand contains fewer than seven cards of that suit.

MAR 2. The October 1987 issue of *John Hopkins Magazine* contained an installment of "Golomb's Gambits" by Solomon Golomb. That month the gambits were entitled "Figures drawn and quartered" and included the following puzzle. Divide the figure below into four congruent pieces. The grid lines indicate the shape but your cuts need not be along the grid. There are four distinct solutions; how many can you find?



Speed Department

Ted Mita offers the following quickie. You notice a billboard containing the word "pop" (without the quotes) on it. Now you stand on your head and look at the reflection of the billboard in a flat mirror. What "word" appears in the reflection?

Solutions

Y2004. The following solution is from Roger Whitman.

$1 = 204^0$
$2 = 40 / 20$
$3 = 2 + 0 + 4^0$
$4 = 20 \times 0 + 4$
$5 = 20^0 + 4$
$6 = 2 + 0 + 0 + 4$
$8 = (2 + 0) \times (0 + 4)$
$16 = 20 + 0 - 4$
$19 = 20 - 4^0$
$20 = 40 - 20$
$21 = 20 + 4^0$
$24 = 20 + 0 + 4$
$38 = 40 + 0 - 2$
$39 = 40 - 2^0$
$40 = 2 \times 0 + 40$
$41 = 20 + 40$
$42 = 2 + 0 + 40$
$50 = 200 / 4$
$60 = 20 + 40$
$80 = (20 + 0) \times 4$

OCT 1. *We begin with a bridge problem from Larry Kells, who wants to know if it is possible to play a complete bridge hand without any discards ever occurring.*

Derek Truesdale found a bridge hand without any discards and supplies an explanation to show that the bidding and play are reasonable.

North
♠ K 7 6 5
♥ A Q 8 7
♦ 9
♣ K 7 6 5

West
♠ 8
♥ 6 5 4 3
♦ 8 7 6 5
♣ 8 7 6 5

East
♠ A 4 3 2
♥ 2
♦ A 4 3 2
♣ A 4 3 2

South
♠ Q J T 9
♥ K J T 9
♦ K Q J T
♣ Q

S	W	N	E
1D	Pass	1H	Dbl
2H	Pass	4H	All Pass

West leads his singleton spade to East's ace. East returns a spade, and West ruffs. West tries to get back to his partner with a club and succeeds as East returns another spade for another ruff. West next tries diamonds with further success, and gets another spade ruff.

It doesn't matter what West leads next. Declarer is able to claim the rest, but as an avid Puzzle Corner reader, he figures he might console his down 3 if the rest of the tricks are played out without discards. He wins the last seven tricks by leading one trump, and crossruffing three clubs and three diamonds. When South points out what happened, nobody appreciates the serendipity more than West, whose pathetic hand won the three setting tricks.

Truesdale notes that the distribution does not even need to be 4-4-4-1 for a no-discard game to theoretically work, but it's harder to come up with a reasonable line of play for many other distributions.

OCT 2. *Tom Harriman has three circles with distinct centers and radii. Each pair of circles has common external tangents (i.e., the circles lie between the tangents) that intersect at a point. He wants you to show that these three points are colinear.*

Victor Barocas sent us the following elegant solution, complete with a carefully drawn, clear diagram.

Let's start with some conveniences that don't destroy the generality. Call the circles 1, 2, and 3, with 1 being the largest and 3 the smallest. The center of circle i is O_i, and a point of tangency on that circle is X_i. Let the point A be the intersection of the external tangents of circles 1 and 2, B the intersection of the external tangents of 1 and 3, and C the intersection of the external tangents of 2 and 3. Since any two points are colinear, we will rotate the system so AB lies on a horizontal line. A diagram is shown below. I have deliberately introduced a slight kink to the external tangents of circles 2 and 3 so C is not colinear with A and B in the picture. The distance between the line containing AB and the point C is Δ. The triangles $A-O_2-X_2$ and $A-O_1-X_1$ are similar since they both contain a right angle AXO and angle XAO is the same for both. Thus, AO_i is proportional to the radius r_i of circle i $(i=1, 2)$. A similar argument applied to the triangle formed by dropping a perpendicular from O_i (of length h_i) shows that h_i is proportional to AO_i, which implies that $h_1/r_1 = h_2/r_2$. Obviously, the same analysis at the point B yields $h_1/r_1 = h_3/r_3$. The same analysis at the point C yields $(h_2+\Delta)/r_2 = (h_3+\Delta)/r_3$, but combining the results from points A and B gives $h_2/r_2 = h_3/r_3$. The only way for both equations to be satisfied is if $\Delta = 0$ (given that $r_2 \neq r_3$), implying that A, B, and C are colinear.

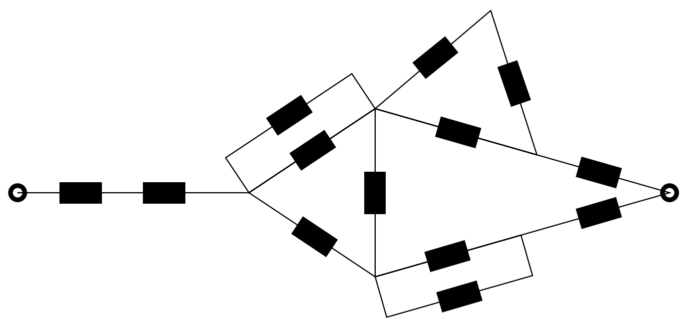
The proposer notes that this problem is known as d'Alembert's Theorem (conjecture, originally) and was first proved by G. Monge in the late 18th century.

OCT 3. *George Blondin has a basement full of perfect 1-ohm resistors. Since the basement is circular, he needs a circuit with resistance π ohms. In a concession to reality, he is willing to accept an error of up to 1 microöhm. What is the fewest number of resistors he needs to employ?*

Several readers used the approximation $\pi = 355/113$ to obtain series-parallel solutions with 26 resistors. The minimal circuit, found by Joel Karnofsky, also uses 355/133 but is not series-parallel, Karnofsky writes.

An application! All my computations of resistances for the Oct. 3, 2003, puzzle made solving the Oct. 3, 2004, puzzle easy. The answer is 13 resistors, with one possible circuit shown here. My computations of all possible resistances for circuits with fewer than 13 resistors produced no values close enough to π.

The resistance of the pictured circuit, computed using Kirchoff's Laws, is 355/113, which differs from π by about .27 microöhms. This fraction is not surprising, since 113 is the smallest denominator of a reduced fraction close enough to π.



Other Responders

Responses have also been received from A. Shagan, P. Balbus, G. Blondin, R. Bowers, J. Bross, B. Byard, S. Chamberlin, C. Dale, R. Dehoney, R. Ellis, R. Giovanniello, J. Harmse, R. Hess, R. Hess, R. Hess, H. Hochheiser, H. Hodara, P. Kebabian, J. Kotelly (dedicated "to a most inspirational mathematics teacher, Mr. F. Gilbert, Boston Latin School, 1950s"), L. Kreymborg, E., L., and N. Lubell, A. Maestri, T. Maloney, R. Marks, P. Martel, R. Merrifield, A. Peralta, S. Resnikoff, K. Rosato, E. Sard, M. Seidel, D. Sherman, E. Signorelli, J. Smith-Mickelson, K. Soch, S. Sperry, H. Thiriez, F. Tydeman, R. Wake, and R. Whitman.

Proposer's Solution to Speed Problem

The name "bob."

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York NY 10003, or to gottlieb@nyu.edu.

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