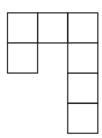
As you may already know, in 2005 *Technology Review* will return to being a monthly magazine, which will impact "Puzzle Corner." After a transition period, my column will appear in every odd numbered month (January, March, ...) with the "yearly problem" included in January, where it began. This increases the number of columns from 5 per year to 6. Timing constraints pre-vented us from achieving this schedule right away; the first "Puzzle Corner" of the year will be in March.

PROBLEMS

DEC 1. Apparently the maxim that ``crime doesn't pay" does not apply to Bridge, at least according to Larry Kells who wants to know hwat is the largest number of tricks that can be gained by one revoke? Assume a two-trick penalty with no further adjustment.

DEC 2. Nob Yashigahara was asked by Junk Kato to cut the letter 7 below (made from 7 squares) into several pieces and re-arrange them into a perfect square. Can you do it?



DEC 3. Norman Spenser would like to know the radius of the inscribed circle for a triangle with side lengths a, b, and c. He would also like to know the radius of the circumscribed circle, but fears that might be harder.

SPEED DEPARTMENT

My queue of speed problems is empty. Perhaps, I should simply drop this department Instead, this month I offer an old quickie that I ran in one of the very first puzzle corners, circa 1966.

If a chicken and a half lays an egg and a half in a day and a half, how many eggs do six chickens lay in six days?

SOLUTIONS

J/A 1. Larry Kells, presumably a law-abiding citizen in normal circumstances, wants us to violate the Law of Total Tricks as violently as possible.

He writes, "Have you heard of the Law of Total Tricks? This is a heuristic which says that the expected number of tricks North-South can take declaring in their longest trump suit, added to the number of tricks East-West can take declaring in their longest suit, is usually equal to the number of trumps North-South have in their suit, added to the number of trumps East-West have in theirs. This is often useful for competitive-bidding decisions.

"My question is, how badly can this law be violated? What is the greatest possible excess of total tricks over total trumps (with best play and defense)? What is the greatest possible deficit? (Assume that for each partnership, the declarer is whichever one would make the most tricks with their suit as trumps, if it makes a difference which side plays it.)"

Tom Terwilliger believes that the law of total tricks does not require either side to be declaring in their longest suit; any suit will do. He sent solutions to both the problem as printed, which appears below, as well as to his interpretation of the problem.

The maximum deviations are +12 (12 more tricks than trumps) and -14.

	♠	
	♥ A K Q J 10 9 8	
	♦ A K Q J 10 9	
♠ A K Q J 10 9 8	♣	♠
♥		♥ 7 6 5 4 3 2
♦		♦ 8 7 6 5 4 3 2
♣ A K Q J 10 9	♠ 7 6 5 4 3 2	♣
	♥	
	♦	
	♣ 8 7 6 5 4 3 2	

Here, N/S has 7 hearts and can take all 13 tricks no matter who is on lead. N simply ruffs the opening lead if black and wins it straight if red. He/she draws trump and cashes all his diamonds. Obviously E/W can do the same thing in spades, so there are 26 total tricks with only 14 total trump.

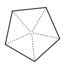
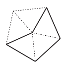
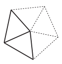
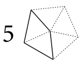
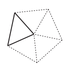
	♠ 4 3 2	
	♥ A K Q J 10	
	♦ 7	
♠ A K Q J 10	♣ 6 5 4 3	♠
♥ 4 3 2		♥ 9 8 7 6 5
♦ 6 5 4 3		♦ 2
♣ 7	♠ 9 8 7 6 5	♣ A K Q J 10 9 8
	♥	
	♦ A K Q J 10 9 8	
	♣ 2	

Here, N/S has 8 spades but can only take 1 trick as long as S declares (W on lead). W draws 5 trump and plays a club to allow E to cash 7 clubs. N/S win the last trick. Obviously E/W can do the same thing in spades, provided W declares (N on lead), so there are 2 total tricks with 16 total trump.

J/A 2. Jerry Grossman has a figure consisting of six points, five of them arranged in a regular pentagon and one more in the center of the pentagon. This figure has 10 line segments—the five sides of the pentagon and a segment from the center to each point on the pentagon.

There are various subsets of five of these segments that can be deleted without disconnecting the figure (for example, one of the segments around the rim and four of the spokes). On the other hand, if we delete some subsets of five of these segments, then we disconnect the figure (for example, if we delete the five spokes, then the center is no longer connected to the points on the rim). Find out how many subsets of each kind there are.

Joel Karnofsky sent us a fine solution that is printed below. In addition he (and Mathematica) believe that, for a wheel graph with n spokes, the number of connected subgraphs resulting from deleting n edges is $T(n) = ((3 + \sqrt{5})/2)n + ((3- \sqrt{5})/)n - 2$.

It is easy to see that a connected graph with 6 nodes and 5 edges must be a tree and, conversely, a disconnected one must contain a cycle. Enumerating the latter, there is 1 of these , 5 of these  (the heavy pentagon can be rotated), 5  (the heavy double triangle can be rotated), 5 x 5  not containing the third shape (rotation gives 5; for each you can darken one of the 5 dotted lines not giving shape 3) and 5 (7 x 6 / 2 - 2)  also not containing the third shape (5 rotations; for each you add 2 of the 7 dotted lines; two of these give the third shape) ; for a total of 131 disconnected subgraphs. Since the total number of ways of deleting 5 of the 10 edges is $10! / (5!)^2 = 252$, the difference, 121, is the number of connected subgraphs.

Here is Mathematica code that computes the number of connected subgraphs of a wheel graph with *n* spokes after deleting any *n* edges. The code generates all relevant subgraphs and counts the ones where the connected component of the center node is all nodes. In the data structures, 0 is the center node and 1, ..., n are the rim nodes.

```
connected[n_]:=Module[{
    edges=Flatten[Array[{{0,#},Mod[{#,#+1},n]+1}&,n],1],
    vars=Array[v,n],graphs },
  graphs=Flatten[Table[
    edges[[vars]],{v[1],2n},Evaluate[Sequence@@Array[{v[#
    +1],v[#]+1,2n}&,n-1]]
    ],n-1];
  Count[connectedComponent/@graphs,Range[0,n]]; ];
connectedComponent[graph_]:=FixedPoint[Union[#,Flatten
[adjacent[#,graph]&/@#]]&,{0}];
adjacent[node_,graph_]:=Cases[graph,{node,n_}|{n_,node}->n];
```

Running this code for n = 3, ... ,10 produces the following:

n	subgraphs	connected	fraction
3	20	16	0.8
4	70	45	0.64
5	252	121	0.48
6	924	320	0.35
7	3432	841	0.25
8	12870	2205	0.17
9	48620	5776	0.12
10	184756	15125	0.08

J/A 3. With this being a U.S. presidential-election year, I feel a dart-throwing problem is in order. So here is one (in a sense, three) from Frank Rubin, who writes, "I recently came across a puzzle intended for young children. They were shown a bull's-eye target, with each concentric ring marked with a different score between 1 and 25, inclusive. The children were asked to find how many darts hit each ring to give a total score of 50. Unfortunately, there were two distinct ways of scoring 50."

Your task is to reconstruct the target, given that there were two distinct ways of scoring (a) every value from 44 to 65, inclusive; (b) the values 19, 50, and 63; and (c) the values 35, 50, and 98. (These are three independent problems.)

The following solution is from Frank Rubin.

Part a. The target cannot have a 1. One by itself only gives one way to score each total. One with any other number N from 2 to 25 gives at least 3 ways to score 50, namely (50)1, (50-N)1+N and (50-2N)1+N.

The target cannot contain a 2. Two with an even number 2N gives (25)2, (25-N)2+2N, and (25-2N)2+(2)2N. Two with an any odd number M=2N+1 from 3 to 13 gives 3 ways to make 65, namely (32-N)2+M, (31-3N)2+(3)M, and (30-5N)2+(5)M. Two and 15 give 3 ways to make 60. Two with an odd number greater than 15 allows only one way to make 44. Two with any two odd numbers M and N from 17 to 25 gives 3 ways to make 50, namely a bunch of 2s and either (2)M, (2)N, or N+M.

Now suppose that the target contains a 3. Three with any multiple of 3 gives 3 ways to make 48. Three with 4, 7, or 10 gives 3 ways to make 60. Three with 5 or 8 gives 3 ways to make 48. This leaves 11, 13, 14, 16, 17, 19, 20, 22, 23, and 25. Three with any one of these numbers leaves only one way to obtain either 44 or 46, so at least two of these numbers are required.

There are 45 pairs of these 10 numbers. Three with any two numbers of the form 3N+1 gives three ways to get 50. Three with any two numbers of the form 3N+2 gives three ways to get 46. Among the pairs of the form (3M+1,3N+2) if either number is 21 or less, then there are 3 ways to make 63.

This leaves only the pairs (22,23) and (23,25). The combination 3, 23, and 25 gives only one way to make 44. The combination 3, 22, and 23 is found to meet the problem conditions. It remains to verify that this is the only solution. The process is just as above.

Parts b and c are solved in the same manner. For part b, a good method of attack is to begin by finding all possible targets involving only numbers 1 to 19 which achieve a total of 19 in two diferen ways, then extending to include 50 and 63. The solutions for parts b and c are (9, 10, 19, 23) and (16, 17, 18, 19) respectively.

BETTER LATE THAN NEVER

2003 OCT 3. Fred Tydeman notes that since we permit "hanging" resistors, then each N-resistor problem has another solution: 0 ohms by letting all the resistors hang and using just the end of the chain. For example, 1 resistor gives two possibilities 1 and 0 ohms.

OTHER RESPONDENTS

Responses have also been received from M. Haggerty, R. Karlsson, D. Katz, N. Markovitz, A. Ornstein, K. Rosato, and E. Signorelli.

PROPOSER'S SOLUTION TO SPEED PROBLEM

24.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York NY 10003, or to gottlieb@nyu.edu.