PUZZLE CORNER

WHAT AN EXCITING SUMMER FOR OUR family! We attended the MIT graduation ceremony in early June (I use the parochial definition of summer: Memorial Day to Labor Day) to see our son David receive an MEng in course I and his then-fiancée receive a BS in biology. If, like me, you shamefully haven't attended an MIT graduation for a long while, you may be surprised to learn that they interweave two lists of students graduating, using a split screen on the large monitors around the great lawn. The pipelining advantage is quite evident. Each parent sees his or her little darling for five seconds, but the thousands of students are "processed" at a rate of one per two and a half seconds.

In a week, the fiancée label will disappear as we attend David and Sarah's wedding on July 17. She is a lovely and accomplished young woman and we are thrilled that David, like his father before him, did so well.

Turning to other matters, since this is the first issue of a new academic year, let me once again review the ground rules. In each issue I present three regular problems (the first of which is normally bridge-related) and one "speed" problem. Readers are invited to submit solutions to the regular problems and in a later columns (not every issue of *TR* contains a Puzzle Corner column) a submitted solution is printed for each regular problem; I also list other readers who responded. For example, the current issue contains solutions to the regular problems posed in May.

I am writing this column in July and anticipate that the column containing the solutions will be due in late November. Please try to send your solutions early to ensure that they arrive before my submission deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Respondents" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously-unsolved problems. The procedure for speed problems is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Comments on speed problems are rarely published.

There is also an annual problem, published in the first issue of each year, and sometimes I republish problems that remain unsolved after they were first presented.

PROBLEMS

OCT 1. We begin with a Bridge problem from Larry Kells, who wants to know if it is possible to play a complete Bridge hand without any discards ever occurring.

OCT 2. Tom Harriman has three circles with distinct centers and radii. Each pair of circles has common tangents that intersect at a point. He wants you to show that these three points are co-linear.

OCT 3. George Blondin has a basement full of perfect 1-ohm resistors. Since the basement is circular, he needs a circuit with resistance p-ohms. In a concession to reality, he is willing to accept an error of up to 1 micro-ohm. What is the fewest number of resistors he needs to employ?

SPEED DEPARTMENT

Matthew Lieff wonders what the following countries have in common: USA, Israel, Saudi Arabia, the Philippines, El Salvador, Colombia, and Bolivia.

SOLUTIONS

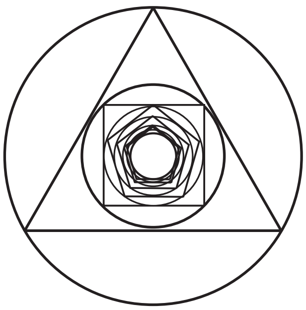
MAY 1. Most of the solutions were in the same spirit as Mark Bolotin's, which follows:

	♠ 9 8 7 6 5 ♥ ♦ ♣ 9 8 7 6 5 4 3 2	
♠ ♥ Q J 10 9 8 7 ♦ Q J 10 9 8 7 ♣ A		♠ A K Q J 10 ♥ A K ♦ A K ♣ K Q J 10
	♠ 4 3 2 ♥ 6 5 4 3 2 ♦ 6 5 4 3 2 ♣	

"The contract is one spade, redoubled. If West leads a red card, South can win the first seven tricks on a cross-ruff with two ruffs in hearts, two in diamonds, and three in clubs. If West leads the club Ace, South cannot afford to ruff. If South ruffs and begins a cross-ruff, he can ruff clubs in his hand and three red cards in dummy. Unfortunately, he is on board and has no more trumps in his hand to ruff back. Instead, South must discard a red card on the club Ace. Now West must switch to a red card and South can proceed with a cross-ruff just as if West had led a club originally.

"I think I know how the auction may have proceeded—if my memory of the rules of contract bridge is correct. Trying to stir up a bad round in his duplicate game, South opened a psychic one spade. West thought he was too strong for an overcall and chose a questionable double. While North was contemplating his bid, East bid out of turn and, consequently, barred his partner from the auction. North redoubled. West, confused about who was barred from the auction, bid out of turn in an attempt to get out of one spade redoubled. Now East was also barred from the bidding. South had no idea where to run and chose to pass."

MAY 2. A solution to this problem consists of three parts: finding the infinite product, showing convergence, and estimating the converged value. As noted by Walter Sun, since every factor in the infinite product is strictly between 0 and 1, a naïve guess would be that the product is zero, which Sun explicitly shows is incorrect. The following solution and figure is from Harold Boas, former editor of the *Notices of the American Mathematical Society*:



"Although I am a bit of a ringer, I can't resist responding to the May 2 problem in your Puzzle Corner in *Technology Review*—it is a problem that I know well.

"First of all, a small correction: the authors of the cited book, *Mathematics and the Imagination*, are Edward Kasner and James Newman (not Neuman as stated in the problem). Now the solution: If an n-gon is inscribed in a circle of radius r, then the midpoint of the chord joining two adjacent vertices of the n-gon has distance $r\cos(p/n)$ from the center of the circle, and this distance is the radius of the circle inscribed in the n-gon. Consequently, the limiting radius of the circles is the product of $\cos(p/n)$ as n goes from three to infinity. Does this product converge? An equivalent question is whether the sum of the terms $\log(\cos(p/n))$ converges. Since $\log(\cos(x))$ is of the same order of magnitude as the square of x when x is a small positive number, the sum converges by comparison with the sum of the squares of the reciprocals of the positive integers.

"The convergence is relatively slow, however, and when Kasner and Newman wrote their book in 1940, getting a good numerical value for the limiting radius may have been nontrivial. Modern computer algebra systems immediately give the numerical approximation 0.114942. In the caption to Figure 125 on p. 311 of their book, Kasner and Newman incorrectly state that the limiting radius is approximately 1/12 the radius of the initial circle. Could they have meant 12 percent?"

MAY 3. This was quite a popular problem; thanks to Nob Yashigahara for proposing it. Bob Byard has sent us a list of all solutions (including those with denominator 1) up to a sum of 163. You can find the list on the column Web page cs.nyu.edu/˜gottlieb/tr. The following solution is from George Blondin:

"If we limit the search to equations in which each numerator is a multiple of the denominator, so that each term of the equation is an integer, there are just 207 valid equations with 63 unique sums:

From 24/6 + 35/7 + 18/9 = 11
To 98/2 + 51/3 + 76/4 = 85
Twenty-three is the sum most often repeated:
12/3 + 78/6 + 54/9 = 23
15/3 + 24/6 + 98/7 = 23
15/3 + 42/7 + 96/8 = 23
18/2 + 63/7 + 45/9 = 23
18/3 + 54/6 + 72/9 = 23
18/3 + 56/4 + 27/9 = 23
18/3 + 72/6 + 45/9 = 23
54/3 + 16/8 + 27/9 = 23

"However, there are other solutions, such as 14/3 + 75/9 + 68/2 = 47, in which the terms include fractions and are harder to count.

"In all, there are 535 equations with 75 unique sums: 11, 13–82, 84–86. and 92; the most common is 34, which occurs 20 times."

BETTER LATE THAN NEVER

2003 OCT 3. Joel Karnofsky has improved his algorithm and now has calculated the number of resistances for up to 15 one ohm resistors. Details are posted at cs.nyu.edu/˜gottlieb/tr.

2003 DEC 2. Joel Karnofsky has written a manuscript on this problem (the three post variant), which I have posted at cs.nyu.edu/˜gottlieb/tr.

OTHER RESPONDENTS

Responses have also been received from R. Ackerberg, S. Anil, E. Asel, T. Barrows, T. Bielecki, R. Bishop, G. Blondin, M. Bolotin, L. Cangahuala, J. Chen, D. Chernoff, A. Colville, J. Cronvich, D. Dechman, R. Ellis, L. Gershun, H. Ghiron, R. Giovanneillo, J. Hardis, D. Herron, H. Hochheiser, H. Ingraham, J. Karnofsky, L. Kells, B. Layton, M. Lieff, T. Maloney, A. Ornstein, G. Perry, E. Sard, H. Sard, R. Segal, E. Signorelli, P. Silverberg, J. Smith, S. Sperry, G. Steele, J. Steele, A. Taylor, T. Terwilliger, R. Wake, G. Waugh, D. Wellington, and E. Yeo.

PROPOSER'S SOLUTION TO SPEED PROBLEM

Each is named after an individual. America: Amerigo Vespucci; Israel: Jacob a.k.a. Israel; Saudi Arabia: Ibn Saud; Philippines: King Phillip V; El Savador: Jesus ("El Salvador" translates to "The Savior"); Columbia: Christopher Columbus; Bolivia: Simon Bolivar.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York NY 10003, or to gottlieb@nyu.edu.