It has been a year since i specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a comfortable supply of regular problems and Bridge problems. Speed problems, however, are in very short supply.

PROBLEMS

MAY 1. Our intrepid correspondent, Larry Kells writes that his bridge friend recently revealed his greatest triumph yet with his new wife. They bid and made a redoubled contract for game and rubber despite their opponents holding all of the aces, kings, queens, jacks and tens, and two of the nines! "Of course our opponents were stupid to let us play it there, but they made no mistake in their defense. There was no way they could beat our contract!" As usual, it was hard to get the details out of him. Can you help?

MAY 2. Matthew Fountain offers us the following problem that first appeared in Kasner and Neuman's 1940 *Mathematics and the Imagination*. In a circle of unit radius inscribe a triangle. Inscribe a circle in the triangle and in this circle inscribe a square. Continue inscribing circles and regular polygons, each polygon with one more side. What is the limiting radius of the circles?

MAY 3. Nob Yoshigahara offers us the "Komachic eleven" problem. Replace the number signs below with the digits \$1 ... 9\$ to yield a valid equation, without using 1 as a denominator:

$$\frac{\#\#}{\#} + \frac{\#\#}{\#} + \frac{\#\#}{\#} = 11$$

Can you find integers other than 11 for which solutions are possible?

SPEED DEPARTMENT

Ken Rosato has a longish speed problem. A standard US flag (you can see one at www.kxtv.com/public/american-flag.htm\-ed [ck]) is altered by adjusting the right boundary of the blue field and the size of the stars so that red, white, and blue cover equal areas. If the length of the flag is L, what is the length of the blue field? If the area of the flag is A, what is the area of each star?

SOLUTIONS

DEC 1. We received a bridge problem from Larry Kells who wonders if it is always possible to make 6NT with ♠AKQ, ♥AKQ, ♦AKQ, and ♣AKJx for different values of x? You are to assume perfect double-dummy play.

I am not a Bridge expert by any means, but after looking at all the received submissions, I believe only the proposer offers a complete solution. Recall that the problem states we are to assume perfect double dummy play and asks if the contract can .i always be made. Hence we assume the worst distribution of the remaining cards. If x is 10 or Q, the answers is clearly yes.

If x is 8 or less, Kells writes: Say LHO has Q9xx and RHO has 10xxx. They can defend against you. They keep 4 clubs apiece until you break the suit. If you lead the J, LHO wins and returns a small club; the 10 forces out one of your honors and LHO's 9 will win a trick. If you lead the 8, RHO can win and return a club, and the Q will win a trick.

The delicate case is when x is 9. Kells begins by defining a 4 card suit to mean 4 or more, 3 card to mean 3 or fewer, side suit to mean not clubs, and shortest suit to permit an arbitrary choice among the shortest if there are more than one. He then writes: If an opponent has the Q or 10 in a 3 card suit, it is easy.

First assume one opponent has Q10xx We execute what I will henceforth call a pressure strategy against that opponent. In this strategy, cash the AKQ of the opponent's shortest suit, which exhausts him. If he now has only 3 clubs left, play the AK and 9 if necessary to set up your J. If he still has 4 clubs, cash the AKQ of his shortest remaining suit. If he now has only 3 clubs, set up the J; otherwise cash your remaining AKQ. This will leave him with only clubs. Lead either the J or 9; if your opponent wins it, he must give you a free finesse for a third club trick. The key to the pressure strategy is that when you have finished cashing out the AKQ of any given suit, your opponent will not have any cards left in that suit, so you can still stop anything he is capable of leading and can safely give him the lead.

If LHO has Qxxx and RHO has 10xxx, do a pressure strategy against LHO. If he ever comes down to 3 clubs, set up your J as before. Otherwise, at trick 10 lead the C-J. If LHO wins, he has to give you a free finesse to set up your 9. ,pp If RHO has Qxxx and LHO has 10xxx, do a pressure strategy against RHO. If he ever comes down to 3 clubs, set up your J as before. Otherwise, what to do at trick 10 depends on what LHO has kept. (He has not necessarily been under pressure.) If LHO has only 3 clubs left, lead the AKJ to trap the 10 and set up your 9. If he still has 4 clubs, lead the 9. If LHO wins, he has to give you a free finesse to set up your J. If RHO wins, he sets up your J immediately.

DEC 2. Richard Hess wants us to solve "the chain problem" due to Bob Wainwright. I have modified it slightly. Consider a chain of length $d = \frac{n\,(n+1)}{2}$ with integer-length links in order 1, 2, 3,..., n. For what n can you wrap the chain in a tight loop around two posts $d/2$ apart so that the posts occur exactly at the break between two links? For example, if $n = 4$, one post is between 3 and 4 and the other post is between 1 and 2. But when $n = 5$, $d = 15$, and integers will not sum to 7.5. What if we have three posts and want the lengths to all be $d/3$?

The summary is that for two posts a solution is possible half the time and for three posts the problem is hard. The following solution is from Kenneth Graves:

Two posts:

We can only construct the desired chain if $d/2 = n(n + 1)/4$ is integral. This occurs for $n \equiv 0$ and $n \equiv 3$, mod 4. In both cases, it is possible to construct one such chain. So the remaining links must form the second chain.

n ≡ 0 mod 4

$$\frac{n(n+1)}{4} = d(n+1)$$
$$= (n+1) + ((n-1)+2) + ((n-2)+3)+\ldots+ ((n-(d-1)) + d)$$
$$= (n-(d-1)) + \ldots + (n-1) + n + 1 + 2 + \ldots + d \text{ [chain 1]}$$
$$= (d+1) + \ldots + (n-d) \text{ [chain 2]}$$

n ≡ 3 mod 4

$$\frac{n(n+1)}{4} = nd$$
$$= (n) + ((n-1)+1) + ((n-2)+2) + ((n-(d-1)) + (d-1))$$
$$= (n-(d-1)) + \ldots + (n-1) + n + 1 + 2 +\ldots+ (d-1) \text{[chain 1]}$$
$$= d + \ldots + (n-d) \text{ [chain 2]}$$

Three posts

Modular arithmetic isn't as helpful here. We need $r = n(n+1)/6$ to be integral, which occurs for $n \equiv 0, 2, 3,$ or $5 \bmod 6$. For only some of these can we construct the chain containing n, and for only some of those can we split the remainder into two equal chains. I wasn't able to analyze which cases work, but I did brute force $n \leq 5{,}000$.

n	*r*	chain 1	chain 2	chain 3
9	15	[9..3]	[4..6]	[7..8]
90	1,365	[76..15]	[16..54]	[55..75]
125	2,625	[118..57]	[58..92]	[93..117]
125	2,625	[103..2]	[3..72]	[73..102]
153	3,927	[136..51]	[52..102]	[103..135]
189	5,985	[156..15]	[16..110]	[111..155]
440	32,340	[361..24]	[25..255]	[256..360]
819	111,930	[672..56]	[57..476]	[477..671]
989	163,185	[825..164]	[165..594]	[595..824]
1,295	279,720	[1,286..730]	[731..1,045]	[1,046..1,285]
1,394	324,105	[1,201..380]	[381..890]	[891..1,200]
1,484	367,290	[1,324..531]	[532..1,008]	[1,009..1,323]
1,701	482,517	[1,466..466]	[467..1,087]	[1,088..1,465]
2,079	720,720	[1,941..939]	[940..1,524]	[1,525..1,940]
2,448	999,192	[2,160..816]	[817..1,632]	[1,633..2,159]
2,925	1,426,425	[2,478..657]	[658..1,812]	[1,813..2,477]
3,024	1,542,600	[2,547..621]	[622..1,853]	[1,854..2546]
4,004	2,672,670	[3,817..1,968]	[1969..3036]	[3037..3816]

DEC 3. Avi Ornstein defines SD(n) to be "the sum of the digits in *n* reduced to a single digit," where *n* is a positive integer. For example:

SD(25): 2 + 5 = 7.

SD(2,598) 2 + 5 + 9 + 8 = 24, 2 + 4 = 6.

For some *n*, e.g., 81, n/SD(n) is an integer. For other *n*, e.g., 83, it is not. What is the longest consecutive sequence of integers for which n/SD(n) is an integer? What are the numbers in the first such longest sequence?

William Craven's clear solution makes the problem look easier than I thought it was. The Rule of 9 (the sum of the digits of any positive integer which is divisible by 9 will also be divisible by 9) adapted for this problem can be stated as "$SD(n)$ equals 9 for all n (where *n* is a positive integer) equally divisible by 9, and equals the remainder when n is divided by 9 for all other *n*."

Since there are only 9 possible results for SD(n), and these results repeat in sequence infinitely, every second occurrence where $SD(n)$ equals 2 will be for n being an odd number and $n/SD(n)$ will not be an integer. The same is also true for $SD(n)$ equal to 4, 6, and 8. On the other hand, values of n where $SD(n)$ equals 1 or 9 will always have $n/SD(n)$ equal to an integer. Thus the maximum potential sequence length is 11.

Every sequence of 11 numbers beginning with a multiple of 2,520 (the LCM of the set [1, 2, 3, 4, 5, 6, 7, 8, 9] of possible results for $SD(n)$) yields results for which $n/SD(n)$ is an integer for every number in the sequence. A few minutes work with a spreadsheet reveals that no sequence beginning with n less than 2,520 is of maximum length. Thus, the first occurrence of this longest sequence is 2,520, 2,521, 2,522, 2,523, 2,524, 2,525, 2,526, 2,527, 2,528, 2,529, 2,530.

OTHER RESPONDENTS

R. Ackerberg, C. Bahne, R. Baum, R. de Marrais, G. Blondin, G. Coram, D. Dechman, S. Gordon, A. Hirshberg, L. Iori, P. Jung, E. Lindblad, F. Lyness, M. Moss, A. Ornstein, J. Pinson, B. Rhodes, K. Rosario, A. Rosenfield, C. Saltsman, J. Steele, T. Terwilliger, and F. Tydeman.

PROPOSER'S SOLUTION TO SPEED PROBLEM

A standard flag has three red stripes (out of seven total) and three white stripes (out of six total) that completely cross the flag, so the area of both red and white in these stripes is 3/13. For the total red area to be 1/3A, the four shorter red stripes must total

$$\left(\frac{1}{3} - \frac{3}{13}\right)A = \left(\frac{13}{39} - \frac{9}{39}\right)A = \frac{4}{39}A$$

Thus, each short red stripe has area $1/39A$. Since a full length red stripe has area $1/13A = 3/39A$, the short red stripe must be 1/3 as long, namely $1/3L$, leaving the blue field a length of $2/3L$. The total area of the red stripes minus the total area of the white stripes is the same as the area of the additional short red stripe, namely, $1/39A$. This must be equal to the area of all 50 white stars, so the area of each star is

$$\frac{1}{50} \times \frac{1}{39}A = \frac{1}{1950}A$$

(As a check, the blue area is the gross area of the field minus the total area of the 50 stars, which is

$$\left(\frac{2}{3} \times \frac{3}{13} - \frac{1}{39}\right)A = \left(\frac{14}{39} - \frac{1}{39}\right)A = \frac{1}{3}A$$

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