

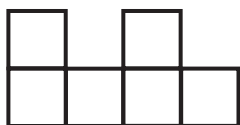
THIS BEING THE FIRST ISSUE OF A CALENDAR year, we again offer a yearly problem, in which you are to express small integers in terms of the digits of the new year (2, 0, 0, and 4) and the arithmetic operators. The problem is formally stated in the Problems section, and the solution to the 2003 yearly problem is in the Solutions section.

PROBLEMS

Y2004. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 4 exactly once each and the operators +, -, x (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 2, 0, 0, and 4 are preferred. Parenthesis may be used for grouping; they do not count as operators. A leading minus sign does count as an operator.

MAR 1 Larry Kells tells us the time his friends encountered their worst luck ever. My friend just told me about the worst luck he has had at the bridge table since he got remarried. He and his wife had all the aces, kings, queens, jacks, and tens, 3 nines and 3 eights. Yet as the cards lay they could not make a grand slam in any suit or no trump from either side of the table. This seemed incredible to me, both because of the sheer power of their hands and because of how much his bridge luck had turned around with his new wife. Was the magic gone? “No,” he said. “She had a premonition that things weren’t breaking well, and stopped at 6NT. This was in a team match, and at the other table they bid 7NT and went down, so we cleared a bundle of IMPs!” So much for his “worst luck.” Can you construct such a deal?

MAR 2. Our last regular problem is from Richard Hess and Robert Wainwright who want you to design a connected tile so that no of them cover at least 93 percent of the area of the hexomino below. The n tiles are identical in size and shape, but may be turned over so that some are mirror images of the others. They must not overlap each other or the border of the hexomino:



SPEED DEPARTMENT

Walter Cluet asks, the fifth letter of the alphabet is to the sixth and the 18th is to the 16 as the 12th is to?

SOLUTIONS

Y2003. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 3, exactly once each, and the operators for addition, subtraction, multiplication, division, and exponentiation? We desire solutions containing the minimum number of operators; and, among solutions that have a given number of operators, those that use the digits in the order 2-0-0-3 are pre-

ferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator.

Things are getting a little better since 2000, but the two zeros are tough. Indeed not until 2013 will we get four distinct digits. Harry Hochhheister and his wife Judith Yanowitz sent us the following solution:

- 1 = 203^0
- 2 = 2+0+0*3
- 3 = 20*0+3
- 4 = 20^0+3
- 5 = 2+0+0+3
- 6 = (2+0)*(0+3)
- 8 = 2^3+0+0
- 9 = 3^2+0+0
- 10 = 30-20
- 15 = 30/2+0
- 17 = 20-0+3
- 19 = 20-3^0
- 20 = 20+0*3
- 21 = 20+3^0
- 23 = 20+0+3
- 28 = 30+0-2
- 29 = 30-2^0
- 30 = 2*0+30
- 31 = 30+2^0
- 32 = 30+0+2
- 50 = 30+20
- 60 = 2*30+0

OCT 1. Larry Kells wants to know, What is your best chance to make 7 spades with

- ♠ A
- ♥ A K Q
- ♦ 5
- ♣ J 9 7 6 5 4 3 2

- ♠ K Q J 10 9 8
- ♥
- ♦ A K Q J 4 3 2
- ♣

The opening lead is a spade, RHO following. Assume there are no inferences to be had from the bidding or the lead, and that the opponents will make no mistakes for the rest of the play.

As we shall see Jerry Grossman remembers his combinatorics well, but for those Bridge fans whose combinatorics are rusty, Bill Kingery recommends www.automaton.gr/tt/en/OddsTbl.htm and thanks Google for the find.

Grossman writes that there are two lines of play to consider: (1) Return to hand with the diamond and pull trumps. (2) Play AKQ of hearts, pitching low diamonds, ruff a club back to hand and pull trumps (there are only four trumps left out).

Choice #1 fails if the diamonds are 5-0. So the probability that #1 fails is the probability of a 5-0 break in diamonds. That

should be about $2 \cdot C(5,5) \cdot C(19,7) / C(24,12)$, which is about 0.037, or 3.7%. (There are 19 nondiamonds and 24 cards in total that the opponents still hold.)

Choice #2 fails if the hearts are 8-2 or worse and it's LHO who will show out of hearts (and has a trump left), or if RHO can ruff a heart in front of you and diamonds turn out to be 5-0.

The probability that hearts are 8-2 with LHO short is $C(10,8) \cdot C(14,4) / C(24,12)$, which is about 0.017; that they're 10-0 or 9-1 with LHO short works out to be negligible, as does the probability that RHO is short in hearts and diamonds are 5-0. So it's maybe two percent total.

Clearly Choice #2 has a greater probability of success and should be adopted. It's about half as likely to fail (although both work over 96% of the time).

At first I thought this problem was actually more interesting, with five trumps still outstanding, but alas there are only four, so if hearts hold up, it is time to ruff a club back to your hand and claim.

OCT 2. Donald Aucamp offers us the "Three Hat Problem." Three logical people—A, B, and C—are wearing hats with positive integers painted on them. Each person sees the other two numbers, but not his own. Each person knows that the numbers are positive integers and that one of them is the sum of the other two. A, B, then C take turns in a contest to see who can be the first to determine his number. In the first round, A, B, and C all pass, but in the second round, A correctly asserts that his number is 50. What are the other two numbers, and how did A determine his was 50?

Several responders praised this problem; thank you Donald Aucamp. In addition to solving Oct. 2, Howard Haber and Aucamp each sent us a more involved variation of this problem, both of which will appear in future Puzzle Corners.

Haber writes, "The key observation is that if two of the three integers are equal, then the person who sees the equal integers knows that his number is twice the same integer.

So, the ratio of the three integers must be 5, 2, 3 for A, B, and C respectively. To see why this is so, consider the first round. A concludes that the numbers are either 1, 2, 3 or 5, 2, 3. At this point, A cannot decide between the two. But, suppose it were 1, 2, 3. Then B would conclude that the numbers were 1, 2, 3 or 1, 4, 3 and could not decide between the two. C then would conclude the numbers were 1, 2, 3 or 1, 2, 1. But C would then reason that the numbers must be 1, 2, 3 because if they had been 1, 2, 1, then B would have known what the numbers were in round one (see key observation above). Since C does not know what the numbers were in round one, when A's turn comes around in round two, he would then conclude that the numbers could not have been 1, 2, 3. Thus, he is now certain that the numbers are 5, 2, 3.

For the problem as stated, just multiply all numbers above by 10."

OCT 3. Fred Gardiol wonders how many different resistances he can obtain by connecting 10 one-ohm resistors.

This problem is harder than I (or several readers) thought.

One cannot generate all $n+1$ -resistor solutions by simply adding a resistor in series or parallel to an n -resistor solution. Tom Terwilliger recognized the difficulties and sent us a partial solution. In particular see his comments for 5 resistors. There is also the question of whether all the values obtained are unique. Luigi Lori consider only the recursively built solutions, but did calculate all the 1,023 values and showed that they are unique.

Terwilliger writes, "The answer is greater than 2047 ($2^{11}-1$), but I don't know how much greater. With one resistor, the only possibility is one. With two, it can be half (in parallel) or two (in series), or also one (leave the second resistor "hanging," using only the first) for three possibilities (2^2-1). With three, you can add the third resistor to each of the three answers above in either series or parallel, for six possibilities, along with one for a single lone resistor, for seven total (2^3-1).

As each resistor is added, we have two possibilities for each previous possibility, plus the lone value of one, so the number obtainable using these combinations only, is 2^N-1 . All such combinations produce unique values. All is well until we get to four. Here there is one additional possibility, namely a square array, measured across the two diagonals. However its value of one has already been used, so there are still only 15 unique resistances that can be obtained.

Now when we get to five, this breaks down, as now we can put three in parallel plus two in parallel for a total of $1/3 + 1/2$ or $5/6$. This, along with $6/5$, obtainable by 3 in series paralleled with two in series, is not present in the list of 31 values obtainable using the above method. There is also $7/6$ and $6/7$, obtained by combining two in parallel with (two in series and one in parallel). Therefore there are 35 possibilities, not 31. For six or more, there are numerous possibilities beyond the basic 2^6-1 obtainable by the first method, and I have been unable to find an algorithm that gives the number of possibilities.

BETTER LATE THAN NEVER

OCT 5D. Larry Rosenbaum noticed that a gigolo is actually a million million billion piccolos.

OTHER RESPONDENTS

Responses have also been received from Auran M, Badavam, D. Berners, G. Blum, A. Bowen, S.S. Brown, R. Byard, R. Chopra, S. Cobb, G. Coram, C. Dale, D. Dechman, M. Garrison, R. Giovanniello, D. Goldfarb, A. Goodisman, J. Grossman, H. Haber, J.E. Hardis, M. Ionescu, L. Iori, R. Kaye, W. Kingery, E. Laipson, T. Lewis, R. Marks, D. Martin, M. Mashaal, A. Ornstein, L. Peters, K.L. Rosato, L.M. Rosenbaum, J. Rudy, J.B.W. Serrao, S.R. Shalom, G. Smith, W. Sun, K. Taylor, A. Ucko, T. Van Laan, J. Walker, F. Webb, R. Whitman, D. Wint, and A. Zobrist.

PROPOSER'S SOLUTION TO SPEED PROBLEM

E:F & R:P as L:I (delete a portion of each letter).

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York NY 10003, or to gottlieb@nyu.edu.