INTRODUCTION

It has been a year since I reviewed the criteria used to select solutions for publication. Let me do so now.

As responses to problems arrive, they are simply put together in neat piles, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses still remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred as are contributions from correspondents whose solutions have not previously appeared. I also favor solutions that are neatly written, typed, or sent via email, since these produce fewer typesetting errors.

PROBLEMS

J/A 1. Larry Kells reports on an ‘‘oddball-sounding remark’’ he overheard at his bridge club. ‘‘That was an awfully risky grand slam you bid! You sure were lucky to get the six-zero trump break you needed to make it....’’ Naturally he was very curious, but never did find out what had occasioned that remark. We are asking for help in solving this mystery.

J/A 2. Ken Rosatto has 12 identical coins and another that looks the same but weighs either more or less. He wants to find the odd ball in three weighings using only a balance.

J/A 3. Rocco Giovanniello has a pyramidal variant of the tetrahedral ‘‘wink jumping’’ puzzle we published last year. The pyramid has four square layers. The top layer has one space, the second has 4, the third 9, and the fourth 16. Each of the spaces except the top is occupied with a wink; the top is empty. The goal is to eliminate all but one wink by a sequence of 3D checker-like jumps.

SPEED DEPARTMENT

David E. Brahm reports that a popular game in Naples FL (at least among his parents) is a dice game called Farkle. A player gets points on his first throw if his 6 dice either a) contain a 1 or a 5, b) form 3 or more of a kind, or c) form 3 pairs. Otherwise the roll is a ‘‘farkle’’ and the turn is over. What is the probability of rolling a farkle if six dice are used?

SOLUTIONS

Mar 1. David Cipolla enjoyed this problem and writes.

The kibitzer is right. With ideal execution by North and South, even after a West lead of the 4 of trumps, the sacrifice could have gone down only 4 and not seven as the players argued.

The requirement is for North and South to give up a trick that they could win, thereby allowing a cross-ruff strategy to become effective. At trick one, North and South must play the 3 and 2, respectively, losing to West's 4. With West still on lead, he must play and win the next three heart tricks while North (or South) drops three diamonds and South (or North) drops three clubs. Note that East cannot win the first round of hearts even if he did not previously discard the 2 and so is unable to gain the lead. When West leads his fourth heart, South ruffs and North drops his last diamond. South then plays any diamond with North ruffing. North then plays a club with South ruffing. This continues until North and South are out of clubs or diamonds. They each have one spade at the end of the cross ruff and either North or South wins that trick.

So East and West win only four tricks: one trump trick and three heart tricks. North and South win nine tricks: one heart ruff, four diamond ruffs, three club ruffs, and a trump trick at the end. Down four.

Mar 2.

This solution can be shortened and/or reformatted nicer

Tom Terwilliger sent us the following detailed solution.

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Weighing #1       1, 2, 3 vs 4, 5, 6       Note that 9 is never weighed!
Weighing #2       1, 4, 7 vs 2, 5, 8
Weighing #3       1,  5  vs  2,  4
Weighing #4       3,  6  vs  7,  8
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You have 72 possibilities of light and heavy coins (9 x 8). In 4 weighings you have 81 possible results (3ˆ4). Therefore if you are clever, a solution should be possible. The trick is to place the proper number of coins on each pan of the scale. Each weighing must divide the above 72 possibilities nearly in thirds, or else you won't be able to come up with a unique solution. I started by listing the results of using various numbers of coins in each pan as follows:

1 coin in each pan: There are 15 ways each pan can be heavy and 42 ways the scale can balance. If you weigh 1 vs 2, then either 1 can be heavy (8 possibilities) or 2 can be light (7 possibilities, not counting the case of 1 heavy and 2 light twice). Since 7 coins are not weighed, the scale will balance anytime both odd coins are in the group of 7, which is 7 x 6 or 42. Clearly this is no good, as after this weighing, there are 42 possibilities left if the scale balances, and there are only 27 results possible on the remaining 3 weighings.

2 coins in each pan: Weigh 1, 2 vs 3, 4. If 1, 2 is heavy then the following are possible:

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1 heavy 3,4,5,6,7,8,9 light 7 possibilities (If 2 is light the scale balances)
2 heavy 3,4,5,6,7,8,9 light 7 possibilities
3 light 5,6,7,8,9 heavy     5 possibilities (Don't count 1 heavy 3 light twice!)
4 light 5,6,7,8,9 heavy     5 possibilities
Total                      24 possibilities
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There are also 24 ways pan 3, 4 can be heavy, and by subtraction 24 ways the scale can balance, i.e.

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5,6,7,8,9 containing both odd coins    5x4 = 20 possibilities
1 heavy 2 light or the reverse                  2 possibilities
3 heavy 4 light or the reverse                  2 possibilities
```

This is perfect, as each weighing divides the possibilities exactly in thirds. Now I spent the better part of a day playing around with how do solve this with 4 weighings each containing 2 coins in each pan. This is not at all symmetric, and I got nowhere. So I wrote a computer program to check all possible combinations and there were none. So I looked at 3 coins in each pan.

Weigh 1,2,3 vs 4,5,6. Pan 1,2,3 will be heavy if:

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1,2,3 heavy 4,5,6,7,8,9 light    6x3 = 18 possibilities
7,8,9 heavy 4,5,6 light          3x3 =  9 possibilities
Total                                  27 possibilities
```

The scale will balance any time the 2 odd coins are in the same group of three, which will happen 6x3 = 18 times.

This is not as good as 24/24/24, but since 4 weighings of 2 coins each didn't work, the solution must involve combining 2 and 3 coin weighings. Now there is an easy way to do two (symmetric) weighings of 3 coins in each pan, namely the rows and columns of a square, i.e.

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#1   1,2,3 vs 4,5,6
#2   1,4,7 vs 2,5,8
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Fortunately there are no more than 9 possibilities for each result. In fact, the scale can never balance both times, and there are exactly 9 ways each of the other 8 results can be obtained, i.e.

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1,2,3 and 1,4,7 pans heavy:
1 heavy 5,6,8,9 light        4 possibilities
3 heavy 5,8 light            2 possibilities
7 heavy 5,6 light            2 possibilities
9 heavy 5 light              1 possibility

1,2,3 heavy 1,4,7 balanced
1 heavy 4,7 light                 2 possibilities
2 heavy 5,8 light                 2 possibilities
3 heavy 6,9 light                 2 possibilities
7 heavy 4 light                   1 possibility
8 heavy 5 light                   1 possibility
9 heavy 6 light                   1 possibility
```

Again, I played around with the other two weighings for a bit and got nowhere, so I made a simple modification to the original computer program to come up with the result at the top. Note its nice diagonal symmetry. Apart from rotations and reflections, it is unique.

The only remaining step is to show that there is no solution if you weigh 3 coins in each pan once and then 2 coins in each pan three times. This was done easily by another trivial modification to the above-mentioned computer program.

BETTER LATE THAN NEVER

Y2002. Avi Ornstein noticed that we mistakenly omitted the parentheses from 10=(20+0)/2.

2002 Oct 3. Tom Harriman and Gunnar Bergman provided very different proofs from the one given. Note that the trigonometry was omitted from the published solution due to space considerations; it was present in Hess's solution. Also I just noticed a typo: the correct formula for θ is $2\sin^{-1}(9/16) + 3\sin^{-1}(1/8)$

OTHER RESPONDERS

Responses have also been received from P.W. Abrahams J. Astolfi D. Bator H.M. Blume, Jr. C.K. Brown C. Cheng D. Cipolla G. Coram J. Dieffenbach I. Gershkoff D. Gross J. Grossman J.E. Hardis T.J. Harriman R. Hess H. Hochheiser J Hoebel S. Hsu S. Kanter D. King S. Klein P. Latham R. Lax B. Layton R. Marks R.D. Marshall L.J. Nissim L. Peters C. Polansky J. Rudy L. Sartori I. Shalom G. Steele C. Swift L. Villalobos G. Waugh

PROPOSER'S SOLUTION TO SPEED PROBLEM

The only way to roll a 6-dice farkle is to form 2 pairs and 2 singletons from the numbers 2, 3, 4, and 6. There are $\binom{4}{2} = 6$ ways to distribute 2/3/4/6 among pairs and singletons and $6 \times 5 \times \binom{4}{2} = 180$ ways to order them. There are $6 \times 180 = 1018$ farkles out of $6^6 = 46656$ possible rolls giving a farkle probability of $1080/46656 \approx 2.3\%$.