Puzzle Corner

INTRODUCTION

It has been a year since I specified the size of the backlogs for the various kinds of problems that are printed. Currently, I have a comfortable supply of regular and speed problems. Bridge problems, however, are in short supply. Peter Kramer was pleased to note that he had solved all the problems in December, "proving that this 54-year-old brain isn't dead yet." He needn't have tested himself. I could have told him that it is at 57 that rapid deterioration begins. At least I think that is right. Hmm, let's see 2002 – 1945 = ? Darn, I keep forgetting where I put that calculator.

PROBLEMS

May 1. The following problem is from Larry Kells, who writes that his friend's latest bridge story about his wife convinced Larry that a lucky charm must hover over their marriage: even the bad turns out to be good.

In one recent game, when vulnerable, he picked up a hand that left him dreading that his luck had run out. He had the worst hand imaginable, ♠ 4 3 2, ♥ 4 3 2, ♦ 4 3 2, ♣ 5 4 3 2. His left-hand opponent dealt and opened 2 hearts, weak. Partner doubled for takeout and inwardly he shuddered with panic, wondering how he could avert disaster. To his relief, his right-hand opponent jumped to 4 hearts, taking him off the hook. Around to partner: to his shock, she leaped all the way to 7 spades! Naturally this was doubled. Down came the ace of clubs on his right, and my friend sheepishly spread his pathetic dummy, certain of impending doom.

Then, suddenly, everything entered the twilight zone. His true love gushed, "Wow! What a magnificent dummy! I was speculating with the 7 bid. With some hands you might have held, it would have been nearly hopeless, but now our chances for making it are greater than 97 percent." And indeed the grand slam rolled home despite the fact that she held only 16 points, and none of the opponents' suits broke evenly. Any idea what the full deal might have been?

May 2. Here is a magic-square problem from Jon Sass. It is quite different from any I have seen before. You are to construct a four-by-four array of positive integers so that 24 different sums total 264. This is not so amazing, especially because the four numbers need not be in a row, column, or on the diagonal. If you turn the magic square upside down, again there are 24 four-number groupings that total 264. Note: turning upside down really means turning the paper 180 degrees in the plane of the paper.

May 3. A celestial mechanics problem from Kern Kenyon: Consider Puzzle Corner as two large equal mass points that revolve about their center of mass. Place a third very much lighter mass point collinear with the first two, but outside (not between) them in such a way that the three masses remain on the line with constant separations as the line rotates about the center of mass. Find the distance of the small mass from the center of mass.

SPEED DEPARTMENT

Seven more lighter-side conversion factors from Sanjay Palnitkar. Identify the following: 500 millionaires; 2,000 mockingbirds; 10 cards; one kilogram of falling figs; 1,000 grams of wet socks; one millionth of a fish; and one trillion pins?

SOLUTIONS

Dec 1. We begin with a bridge problem from Larry Kells, who writes, "I had an unfortunate occurrence at my bridge club: I got a hand that was so strong, I needed only to know my partner's holding in diamonds to know whether to bid grand slam. We have a bidding convention for that very purpose, and I found out that my partner had ♦ A K Q, so I confidently bid 7NT. The spade king was led, dummy appeared, and by freakish misfortune, the cards turned out to be aligned so that I went down! Then I realized that if my partner had held ♦ A Q only (or A K), 7NT would have been impregnable against any possible distribution of the rest of the cards among the other three hands. I have already told you all you need to know to determine my own exact hand and my partner's. What were they?

David Jenkins, who is clearly quite experienced at confidently bidding 7NT, had little trouble with this problem and writes, "To confidently bid 7NT, declarer must have all suits covered and no obvious losers. Therefore, declarer had the aces of spades, hearts, and clubs, plus 10 diamonds headed by the jack. We know that dummy had ♦ A K Q. Because the contract was not made, the declarer must have been stuck in the dummy with no way to return to his hand. This can happen only if dummy's other 10 cards are spades headed by the jack. At some point in the play, declarer must play diamonds and once in dummy must surrender a trick to the queen of spades. If dummy had had only ace and another diamond, the opponents would have had the 13th diamond, which would fall to the ace.

If dummy's 13th card is the queen of spades, declarer wins the first two tricks with the ace of spades and another ace (discarding a diamond), crosses to dummy with a diamond ace, and takes 10 more spades. If the dummy's 13th card is either a heart or club, declarer wins the opening lead with an ace, crosses to dummy's ace of diamonds, returns to an ace in the closed hand, and runs the diamonds for a total of one spade, one heart, 10 diamonds, and one club.

Note that the 7NT contract is a laydown as long as dummy is not void in two suits because then declarer can return to the closed hand and run the good diamonds with any lead. Also, if dummy holds the spade queen, spades can be run to make 7NT. Finally, on the original deal, any lead other than a spade leaves an entry to declarer to run the diamonds and make the contract."

Dec 2. Kern Kenyon has three arbitrary mass points—A, B, and C—located in space. He wants you to show that the following three related points are collinear: Point A, the center of mass of B and C, and the center of mass of A, B, and C.

Walter Sun makes it look easy. First, if A, B, and C are collinear, then the center of masses of any of their combinations will be along that same line, and thus the desired three points will be collinear. So assume they are not collinear. Any three noncollinear points in space define a plane. On this plane, without loss of generality, let A be at the origin and choose the x-axis so that the center of mass of B and C is on it. Thus, we have A = (0,0), B = (x1,*b*), and C = (x2,*c*), where, to ensure that the center of mass of B and C is on the x-axis, we must have mass(B) * *b* + mass(C) * *c* = 0.

What remains is to prove that the center of mass of A, B, and C is on the x-axis. But the y-coordinate of this center of mass is mass(A) * 0 + mass(B) * *b* + mass(C) * *c* = 0 + 0 as desired.

Dec 3. John Regan has an interesting probability question, couched, as is often the case, in terms of dice. If two contestants each roll one six-sided die, the odds are one in six that the values are equal. If they each roll two dice instead, the odds are 146 in 1,296 that the sums of the two dice are equal. What happens if they each roll *n* dice, for *n* large? What if the dice have *m*—instead of six—sides? We are assuming fair dice.

I confess to being unsure about this one because I am not an expert on the mathematics involved. I received a number of solutions that appear correct, but they give different answers. Since there are approximations involved, perhaps there is agreement. The following solution from Craig Wiegert falls within the plurality camp and seems correct to me. Comments are welcome.

Let $p_n(s)$ be the probability that the sum *s* occurs when *n* fair *m*-sided dice are rolled. The problem asks for the probability $P(n)$ that two people roll the same sum, which is $\sum_s p_n^2(s)$ We know that $p_1(s)$ is $1/m$ for $1 <= s <= m$ and 0 otherwise. For $n > 1$, we can define $p_n(s)$ recursively:

$$p_n(s) = \sum_t p_{n-k}(t)\, p_k(s-t).$$

In other words, the probability of rolling *s* with *n* dice is the probability that the first *n-k* dice total *t* and the remaining *k* dice make up the difference, summed over all possibilities *t*.

We wish to calculate the mean and variance of the probability distribution $p_n(s)$, which are defined as $\mu_n = \sum_s s p_n(s)$ and $\sigma_n^2 = \sum_s (s-\mu_n)^2 p_n(s)$. Since *s* is the sum of *n* independent random variables s_i (each with probability distribution $p_1(s)$), the mean of *s* is the sum of the means of the s_i; similarly the variance is the sum of the *n* individual variances. Using $\sum_{i-1}^{m} i = m(m+1)/2$ and $\sum_{i-1}^{m} i^2 = m(m+1)(2m+1)/6$, we determine that $\mu_1 = (m+1)/2$ and $\sigma_1^2 = (m^2-1)/12$. Thus $\mu_n = n\mu_1 = n(m+1)/2$ and $\sigma_n^2 = n\sigma_1^2 = n(m^2-1)/12$.

Because of the symmetry of the initial distribution $p_1(s)$, it is easy to show that $p_n(s)$ is symmetric about the mean μ_n. We can use this fact and the recursion relation for $p_n(s)$ to simplify the expression for $P(n)$:

$$P(n) = \sum_s \text{ from } p_n^2(s) = \sum_s p_n(s)p_n(2\mu_n - s) = p_{2n}(\mu_{2n}).$$

Finally, we appeal to the central limit theorem, which says that for large *n*, $p_n(s)$ will approximate a Gaussian distribution with mean μ_n and variance σ_n^2:

$$p_n(s) \approx \frac{1}{\sigma_n\sqrt{2\pi}}\, e^{-(s-\mu_n)^2/2\sigma_n^2}.$$

So the probability two people will roll the same sum of *n* dice is

$$P(n) \approx \frac{1}{\sigma_{2n}\sqrt{2\pi}} = \sqrt{\frac{3}{\pi n\,(m^2-1)}}.$$

Even for small *n* this is a good approximation to the actual probability. The largest discrepancy is about 13 percent for $m = 2$, $n = 1$; for $m > 2$, the approximation is never more than 4.2 percent off (maximum occurs at $n = 2$). The difference between the exact and approximate results is of order $1/n^{3/2}$.

BETTER LATE THAN NEVER

2002 Jul 1. Victor Barocas submits the following cute comments about Jul 1 and its predecessors: "Larry Kells's friend's new wife is very clever, but she is not a very good duplicate-bridge player. Making four hearts doubled scores 790. However, doubling the opponents' three-heart bid (they have nowhere to run) and setting it at six is worth 1,700. A similar scenario arose previously when the new wife overcalled and made a contract in the suit of the old wife's preempt—a double and a big set would have been less flashy but worth a lot more. I realize the purpose is to pose an interesting exercise, but I must defend the real victim: the friend, who is so blind with love that he doesn't realize he keeps getting bottom boards."

Jul 2. The computer solution given in December lacked an important proof. James Russell closes the gap by providing an alternative analytic solution, which appears on my Web site: allan.ultra.nyu.edu/~gottlieb/tr/.

OTHER RESPONDENTS

Responses have also been received from: Aurion, C.J. Boardman, R. Britto, M.J. Chartier, T.Y. Chow, G. Coram, E. Friedman, J. Grossman, R. Hanau, T. Harriman, R.I. Hess, H. Hodara, E. Kaplan, J.O.M. Karlsson, J. Kenton, P. Kramer, D.C.P. LaFrance-Linden, J. Mahoney, L.J. Nissim, J. Paulsen, B. Rhodes, K.L. Rosato, E. Sard, T.B. Sauke, C.M. Tenney, J. Walker, C. Wiegert, and T. Zimmerz.

PROPOSER'S SOLUTION TO SPEED PROBLEM

One seminary, two kilomockingbirds, one decacards, one Fig Newton, one liter *Hosen*, one microfiche, one terrapin.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York, NY 10003, or to gottlieb@nyu.edu.