Puzzle Corner

INTRODUCTION

For the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 0, 3) and the arithmetic operators. The problem is formally stated in the Problems section, and the solution to the 2002 yearly problem is in the Solutions section.

PROBLEMS

Y2003. How many integers from 1 to 100 can you form using the digits 2, 0, 0, and 3 exactly once each and the operators for addition, subtraction, multiplication, division, and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions that have a given number of operators, those that use the digits in the order 2-0-0-3 are preferred. Parentheses may be used for grouping; they do not count as operators. A leading minus sign does count as an operator.

Mar 1. I wonder whether Larry Kells was a Course I major: he is such an expert on bridge(s). Today he reports on an argument that arose at his bridge club over the following deal:

	♠ K J 9 7 5 3	
	♥	
	♦ 9 7 5 3	
	♣ 6 4 2	
♠ 4		♠
♥ A K Q J 10 9 8 7 6 5 4 3		♥ 2
♦		♦ A K Q J 10
♣	♠ A Q 10 8 6 2	♣ A K Q J 10 9 8
	♥	
	♦ 8 6 4 2	
	♣ 7 5 3	

Neither side was vulnerable. After South's weak 2 of spades opening, East-West bid up to 7 of hearts and made it for +1,510. Afterward, one of the kibitzers expressed an opinion that a 7 of spades sacrifice would have been worthwhile for North-South. The four players all objected that as long as West led his trump, 7 Spades was obviously doomed to go down seven for –1,700, more than the value of the successful grand slam. The way they were shouting disparagement at the kibitzer's lack of any kind of basic bridge sense, he couldn't get another word in edgewise. Who was right?

Mar 2. Andrew Russell offers an interesting variant of an old problem. You have nine coins all of which originally weighed the same. Some material has been removed from one coin and added to another, so that the total weight is unchanged. You have four weighings on a balance scale. Can you determine, in advance, four weighings that permit you to determine the lighter and heavier coin? ("In advance" means no weighing can depend on the results of previous weighings.)

SPEED DEPARTMENT

J.E. Prussing enjoys old-fashioned analog clocks. He wants you to determine the time between successive alignments of (a) the hour and minute hands, (b) the minute and sweep-second hands, and (c) the hour and sweep-second hands.

SOLUTIONS

Y2002. The low point was Y2000, but we are still in tough times. I continue to forbid 0^0 since for x 0, $0^x = 0$ $1 = x^0$. Richard Marks, Avi Ornstein, and C. Dale all agree on the following:

1 = 20/20	19 = 20-$2^0$0
2 = 20*0+2	20 = 20+0*2
3 = 20^0 0+2	21 = 2^0 + 20
4 = 2+0+0+2	22 = 20+0+2
10 = 20+0/2	40 = 20+20
18 = 20+0-2	100 = 200/2

Oct 1. Larry Kells wonders what is the highest high-card point (HCP) count one can hold, including all four aces, such that there is a distribution of cards among the other three hands for which 3NT (no trump) is set with best play on both sides.

Guy Steele gave an amazingly thorough analysis including several difficult corner cases. Sadly, space limitations only permit an abbreviated version. Readers are encouraged to see the full solution at allan.ultra.nyu.edu/~gottlieb/tr. Steele first proves that if either declarer or dummy holds 32 points, 3NT is unstoppable. Then, he exhibits a hand in which either declarer or dummy may hold 31 points and defenders can set 3NT against any strategy.

There are 57 ways one hand can hold 32 HCP or more while holding all four aces. Inspection reveals that such a hand has nine top tricks in all cases except four. In the three cases AKQJ AKJ AQJ AQJ, AKQ AKQ AQJ AQJ, and AKQ AKJ AKJ AQJ, the hand has eight top tricks and a ninth is easily developed by the strategy of playing the J of any suit headed by AQJ at the first opportunity.

The last case is AKJ AKJ AKJ AKJ. If declarer holds these, any opening lead gives a free finesse for the ninth trick. The unlikely case that dummy holds these cards is difficult. See the Web site for its solution.

Now consider the following defense defeats 3NT for the 31 HCP hand shown below. On the first trick, defenders play the S2 and S5. If declarer ever leads the SK, W plays the S3 and E the SQ. On any other lead from declarer, a defender wins it if possible, as cheaply as possible; follows suit if possible, as cheaply as possible; or discards. W discards as cheaply as possible, alternating between the minor suits, while E discards a low heart the first time and the fifth time, and otherwise discards the second-cheapest spade (preserving the S4).

When declarer gives up the lead, there are two cases. If declarer has not yet played the SK, W plays the S3 and E the SQ, forcing declarer to win with the SK. When declarer gives up the lead again, there are two subcases. If declarer has played at most five minor-suit cards, then E has at least three spades left; playing the SJ (W leads or follows with the 8) followed by the S10 and S4 wins three more tricks. But if declarer has played all six minor-suit cards, then W can win the S8 (E playing the S4), D10, and C10.

If the SK has already been played when declarer first gives up the lead, there are two subcases. If E has four or more spades, they are good. If E has fewer spades, then E made at least four discards, and there are two subsubcases. If declarer played the DA, DK, CA, and CK before giving up the lead, then W can win five tricks with the S 8 (E playing the S4), DQ, D10, CQ, and C10 (if declarer exited with a heart, he goes down two). But if declarer played only three of the top minor-suit honors, the fourth discard by E must have been on the DJ or CJ. Because declarer did not give up the lead by playing a heart, declarer has played at most two hearts, therefore W has discarded at most twice, once from each minor suit, and so holds both the Q and 10 in the suit of the J that declarer led. W wins the J with the Q, plays the 10 (E discarding the H Q), and E overtakes the S 8 to win three spade tricks.

	♠	
	♥ 6 5 4	
	♦ 9 8 7 6 5	
	♣ 9 8 7 6 5	
♠ 8 3 2		♠ Q J 10 9 7 6 5 4
♥		♥ Q 10 9 8 7
♦ Q 10 4 3 2		♦
♣ Q 10 4 3 2	♠ A K	♣
	♥ A K J 3 2	
	♦ A K J	
	♣ A K J	

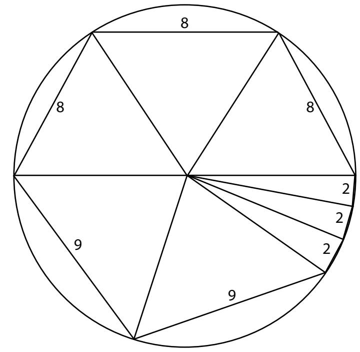
Oct 2. Frank Rubin, an honorary citizen of Frankonia, writes to us about its currency. The island of Frankonia issued three commemorative silver coins between 1802 and 1899, inclusive. The denomination of each coin is the same as the year in which it was issued. The unusual denominations have led to a number of coin puzzles. Two such puzzles are: Find the unique way to make 861 franks using 10 silver coins and What is the only way to make 223 franks with 16 silver coins? Solve these two coin puzzles.

Joel Karnofsky, with the aid of an unnamed computer, found 11 possible solutions, which are believed to be exhaustive. He writes, "I interpreted the problem to mean that the silver coins are denominated in franks, and their denominations are integers in the range 2–99. Presumably there was supposed to be exactly one set of denominations consistent with solving the two coin puzzles or at least only one pattern of coin counts, but a computer search found 11, shown below. In each solution, the first three numbers are the denominations, the next three are coin counts (a coin puzzle solution) summing to 10 and giving a total of 861 with these denominations and the last three are coin counts summing to 16 and giving a total of 223 franks. If my program is correct, each set of coin counts is unique with these properties."

9, 69, 88; 0, 1, 9; 15, 0, 1	4, 77, 90; 0, 3, 7; 14, 1, 1
5, 33, 92; 0, 1, 9; 13, 2, 1	7, 33, 92; 0, 1, 9; 14, 1, 1
3, 15, 94; 0, 1, 9; 8, 7, 1	7, 15, 94; 0, 1, 9; 12, 3, 1
6, 25, 95; 1, 0, 9; 13, 2, 1	6, 44, 95; 1, 0, 9; 14, 1, 1
3, 85, 96; 0, 9, 1; 14, 1, 1	9, 88, 98; 1, 3, 6; 15, 1, 0
11, 58, 99; 1, 1, 8; 15, 1, 0	

Some simple ideas that sped up the search: the smallest denomination must be less than the average 223/16 and the highest denomination must be greater than the average 861/10. The greatest common denominator of the coin counts (and, separately, the denominations) must divide the desired total.

Oct 3. Matthew Fountain likes to inscribe octagons inside circles. He especially enjoys doing this when the radius of the circle and all the sides of the octagon have integer lengths. What is the smallest radius for which this is possible?



The smallest solution Richard Hess could find was R = 8, which has three solutions. The simplest is shown above. Tom Harriman and a calculator showed that no smaller solution is possible. Hess notes that the upper half of his figure is clearly covered by the three equilateral triangles and that $\theta = 2^{-1}(9/16) + 3\ sin^{-1}(1/8)$ is the sum of the half angles of the triangles meeting at the center. Using trigonometry one sees that *sin θ* is *exactly* 1. Thus sum of the half angles is 90^0, and hence triangles exactly fill the remaining semicircle.

BETTER LATE THAN NEVER

2002 Mar 2. G. Quinn notes: sine should be $\sqrt{5/3}$, not $\sqrt{5/3}$.

OTHER RESPONDENTS

Responses have also been received from D. Ballantine, C. Boardman, T. Coradetti, D. Dechman, P. Drouilhet, S. Feldman, H. Fletcher, R. Giovanniello, H. Ingraham Jr., L. Iori, M. Lindenberg, J. Pinston, K. L. Rosato, E. W. Sard, E. Sheldon, E. Signorelli, R.A. Wake, and A.G. Wasserman.

PROPOSER'S SOLUTION TO SPEED PROBLEM:

(a) 12/11 hours, (b) 60/59 minutes, and (c) 720/719 minutes.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, 7th Floor, New York, NY 10003, or to gottlieb@nyu.edu.