Puzzle Corner

INTRODUCTION

Let me begin by correcting a typo in the last issue. The URL for parts of "Puzzle Corner" that could not fit in the allocated two pages of *Technology Review* is http://allan.ultra.nyu.edu/~gottlieb/tr.

I am writing this column in late June and anticipate that the 2003 March column will be due in late November. Please try to send your solutions early to ensure that they arrive before my submission deadline. Late solutions and comments on published solutions are acknowledged in the "Other Respondents" section. Corrections and additions are in the "Better Late Than Never" section.

PROBLEMS

Oct 1. Larry Kells wonders what is the highest high-card point total one can hold, including all four aces, such that there is a distribution of cards among the other three hands for which 3NT is set with best play on both sides.

Oct 2. Frank Rubin, an honorary citizen of Frankonia, writes to us about their coinage. The island of Frankonia issued three commemorative silver coins between 1802 and 1899, inclusive. The denomination of each coin is the same as the year in which it was issued. The unusual denominations have led to a number of coin puzzles. Two such puzzles are, "Find the unique way to make 861 franks using 10 silver coins," and "What is the only way to make 223 franks with 16 silver coins?" Solve these two coin puzzles.

Oct 3. Matthew Fountain likes to inscribe octagons inside circles. He especially enjoys doing this when the radius of the circle and all the sides of the octagon all have integer lengths. What is the smallest radius for which this is possible?

SPEED DEPARTMENT

A quickie from Eugene Gath designed for those familiar with duplicate bridge. What are the next three terms in the sequence 1, 2, 3, 4, 6, 22, ... ?

SOLUTIONS

May 1. Larry Kells wonders what is the fewest number of tricks a partnership can win in a full rubber and still win the rubber (using contemporary rubber bridge scoring).

The answer is, surprisingly, two (ignoring some technicalities below). Guy Steele's solution follows. In addition to finding some rule interpretations that permit an answer of 0, Steele's solution is commendable for showing that, in the spirit of the problem, not only are two tricks sufficient, but one trick is not.

First of all, in a club rubber bridge tournament, the arbiter has broad powers to adjust the score. If NS were to bid and make 7S and then 7H, and EW were immediately to point out to the arbiter that N had signaled S to bid spades by pelting E with a chocolate cream pie (and strawberry for hearts), the arbiter might well adjust the score so that EW win the rubber without winning any tricks.

Let's agree to keep the arbiter out of it. Now we must consider revokes. Suppose EW bid 2H and win no tricks during the play, but NS revoke four times, once in each suit. Law 64 of the 1993 Laws of Contract Bridge speaks of "the offending player" as possibly winning the trick. The penalty is that that trick and perhaps another "won" by the offending side are "transferred" to the nonoffending side. Law 77 states, "A trick transferred through a revoke penalty is reckoned for all scoring purposes as though it had been won in play by the side to which it had been awarded." Thus as many as eight tricks "won" by NS may be "transferred to" (not "won by") EW and are scored "as though" EW had won them, so they make their contract. After four such hands, EW win the rubber without winning any tricks.

But perhaps all this is not in the spirit of the problem as posed. Let us agree to consider "transferred" tricks to have been (ultimately) "won" by the nonoffending side. Then the answer appears to be "two tricks."

Existence proof: First hand, NS bid 5D or 5C and make 7, scoring 140. Second hand, NS bid 7NT, doubled and redoubled, down 2; EW score 1,000. Third hand, NS bid 5D or 5C and make 7, scoring 140. NS score 700 rubber bonus. Final score NS 980, EW 1,000.

There are other ways to do it as well, all of them somewhat less plausible, involving EW earning points for honors:

NS bid 3NT, make 7; NS score 220
NS bid 7NT redoubled, down 2; EW score 1,000 and 150 honors
NS bid 3NT, make 7; NS score 220 plus 700
Final score NS 1,140, EW 1,150

or (even less plausible but, hey, it's only a puzzle):

NS bid 4S, make 7; NS score 210
NS bid 7NT redoubled, down 1; EW score 400 plus 150 honors
NS bid 7NT redoubled, down 1; EW score 400 plus 150 honors
NS bid 4C, make 7; NS score 140 plus 700 rubber bonus
Final score NS 1,050, EW 1,100

Now we need a proof that it can't be done by winning just one trick. EW must win (in our agreed-upon sense) at least seven tricks to fulfill a contract; therefore they must win the rubber without ever fulfilling a contract. Therefore NS will get a 700 rubber bonus. It does not ever benefit EW to be the declaring side, because if EW wins at most one trick, the undertrick penalty will be at least 300 (down six) and EW can gain at most 150 honors, for a net loss; EW can do better by using their one won trick for an undertrick penalty against NS. The only points available in the scoring system to a side that doesn't declare are undertrick penalties and honors. If EW win only one trick, then NS can fail to fulfill their contract on at most one hand. On such a hand EW can earn at most 550 points (400 undertrick penalty and 150 honors). Comparing that to the 700 rubber bonus, that leaves EW down 150 points. But on any hand for which EW take no tricks, they can gain at most 10 points; the only NS contracts for which the honor bonus available to EW exceeds the points won by NS are minor-suit contracts below slam, for which NS win 140 and EW earn 150 (see below for how this is possible). However, EW can pull this stunt at most 10 times before the rubber is over (because NS earn at least 20 points below the line each time, and after 10 times will have won two games). So EW cannot make up the 150-point deficit on hands where they take no tricks. Therefore EW cannot win the rubber while winning only one trick.

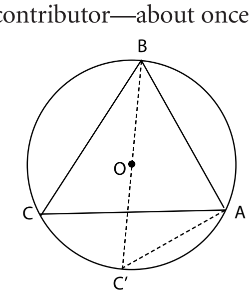
You might think it is impossible to hold 150 honors in a trump contract and take no tricks, because the ace of trumps must take a trick; but that assumes no irregularities. Perversely, EW can hold 150 trump honors while winning no tricks (in our agreed-upon sense) by committing a number of revokes.

Suppose NS were to bid 1C with E holding 150 honors. During the hand, E could revoke on three different tricks, in three different suits, by playing a trump honor, taking the trick, when he could have followed suit. He later takes two more tricks with the other two trump honors. At the end of play, all five tricks are transferred to NS as revoke penalties, and we have agreed that this means that NS "won" the tricks. As a result, EW won no tricks, NS score 140 (20 below the line, 120 above) and EW have a net gain of 10 points.

Actually, there is a way for E or W to hold 150 honors in a trump contract and take no tricks without committing any irregularity. All he needs to do is concede all remaining tricks before he has won any! If neither declarer nor his partner objects (even after he has indicated that he is entitled to the honors bonus!), by Laws 70 and 71 the concession stands.

May 2. Norman Spenser has a triangle with sides of length *a*, *b* and *c*. What is the radius *R* of the circumscribing circle?

The following solution is from Farrel Powsner, who writes that he is "a regular contributor—about once every seven years!"



We are given a triangle *ABC* and a circumscribed circle with center *O*. From any vertex of the triangle, say *B*, draw the diameter *BOC′* where *C′* is on the circle and then draw *AC′*. Angle *BAC′* is a right angle since it intercepts the arc *BC′*, which is a semicircle. Angles *C* and *C′* are equal since they both intercept the arc *AB*. Hence

$$\sin C = \sin C' = \frac{AB}{BC} = \frac{c}{2R}$$

Let *K* be the area of triangle ABC and recall that, for any triangle, $K = \sqrt{s(s-a)(s-b)(s-c)}$ and $K = \frac{1}{2}b(a \sin C)$ where $s = (a+b+c)/2$ is the semiperimeter. Equating the occurrences of sin *C* we get $R = abc / 4K$ and applying the semiperimeter equation yields our final result

$$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

May 3. Steve Omohundro and Peter Blicher have performed a detailed analysis of the availability of Chicken McNuggets at McDonald's. They know that McNuggets come in packs of 6, 9 and 20 and wish to determine the largest number of McNuggets you can't buy (presumably so that they can order precisely that many).

Ken Rosato sent us the following neat solution. The greatest number of McNuggets that cannot be pieced together from packs of 6, 9 and 20 is 43. Any number divisible by 3, other than 3, can be pieced together from multiples of 6 and/or 9. Since 43 is not divisible by 3, it would need to be either 20 or 40 [2 × 20] greater than a number divisible by 3. 23 is not divisible by 3, and 3 is too small to be pieced together from 6s and 9s. So 43 cannot be made from 6s, 9s and 20s.

That leaves proving that all numbers greater than 43 can be made from 6s, 9s and 20s. Since 36 = 9 + 9 + 9 + 9, 38 = 20 + 9 + 9, 40 = 20 + 20, and all greater even numbers can be formed by adding a multiple of 6 to 36, 38, or 40, all even numbers 36 or greater can be formed. Adding 9 to any even number results in an odd number, so all odd numbers 45 or greater can be formed. Thus 43 is the largest number that cannot be formed by adding 6s, 9s and 20s.

OTHER RESPONDENTS

Responses have also been received from R. Ackerberg, T. Barrows, S. I. Berkenblit, M. Brill, H. Bruck, L. A. Cangahuala, J. Chandler, D. Diamond, R. Ellis Jr., S. Feldman, E. Friedman, J. Grossman, J. E. Hardis, R. I. Hess, R. Hoffman, M. Hovey, L. Iori, J. Kenton, T. Kok-Choon, B. Kulp, P. Latham and his Algebra I class, B. Layton, M. Lehman, E. Lindblad, M. K. Lindenberg, J. Mahoney, M. Majewski, N. Markovitz, Z. McGregor-Dorsey, J. Mirzoeff, M. Moss, S. Nason, P. E. Newton, A. Ornstein, E. Signorelli, K. L. Rosato, E. Sard, L. Sartori, J. B. W. Serrao, I. Shalom, D. Sherman, R. G. Sinclair, J. M. Steele, L. Ting, A. M. Ucko and C. Wiegert.

PROPOSER'S SOLUTION TO SPEED PROBLEM

29, 31, 32. Ten times the given sequence is 10, 20, 30, 60, 220, which are the first unachievable scores in duplicate bridge (that are divisible by 10). The next such scores are 290, 310, 320.

Send problems, solutions and comments to Allan Gottlieb, New York University, 715 Broadway, 7th floor, New York NY 10003, or to gottlieb@nyu.edu.